Discussion on Cooper and Corbae's "Dynamic Assignment"

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The General Approach

- Discrete allocation problem: I goods to be allocated across J agents
- a_i: quality of object $i = 1, \ldots, I$.
- ξ_j : user characteristic $j = 1, \ldots, J$.
- $y_{ij} = \phi\left(a_i, \xi_j\right)$ match value of allocating i^{th} input to j^{th} user.
- $Y = \sum_{i,j} y_{ij}$: aggregate return (output)

General Approach

Static Assignment Problem

- $z_{ij} = 1$ denotes input i given to agent j
- feasibility assumptions

  $z_{ij} \in \{0, 1\}$: discrete choice

  $\sum_i z_{ij} \leq 1$ for each $j = 1, \ldots, J$: one input per user (discreteness)

  $\sum_j z_{ij} \leq 1$ for each $i = 1, ..., I$: one user per input (scarcity)

- Planner's Problem

$$\max_{\{z_{ij}\}} \sum_{i,j} z_{ij} \phi \left(a_i, \xi_j \right) \text{ subject to feasibility}$$

General Approach

Examples of match payoff functions

- supermodular matching: $\phi(a, \xi) \equiv \min(a, \xi)$

  example: machines and workers

  implies assortative matching; efficient allocations will have inequality

- submodular matching: $\phi(a, \xi) = \max(a, \xi)$

  example: durable good replacement

  allows redistributive allocations; replace oldest car

General Approach

Comments

- Could have sharper link between goods and users.
  - y_{ij} might be a primitive

- Hard problem, given difficulties in characterizing optimal choice of Z due to discrete choices.
  - are discrete choices natural to the dynamic assignment problem?
  - would be good to have a general approach to characterize solution
  - lotteries could be useful

Durable Goods Example

Environment

- household valuation of car services: ξ_j, $j = 1, \ldots, J$,
- car of vintage i services: $s_i = \gamma^{-(i-1)}$
- match quality: $\phi\left(a_i, \xi_j\right) = s_i \xi_j$ (where $s_i = a_i$)

- c_j: nondurables. $\mathbf{z}_j = \left[z_{1,j}, \ldots, z_{i,j,\ldots}\right]$: durables assignment.

$$U\left(c_j, z_j, \xi_j\right) = u\left(c_j\right) + \sum_{i=1}^{\infty} z_{ij} \xi_j s_i$$

- e: number of new cars produced at relative price ϕ

$$\phi e + \sum_{j=1}^{J} c_j + \Big(K' - (1-\delta) K\Big) \leq AK^{\alpha}$$

Durable Goods Example

Planning problem

$$V(A, K, f) = \max_{e,\ K',\ \{c_j, z_j\}_{j=1}^{J}} \sum_{j=1}^{J} U\left(c_j, \mathbf{z}_j, \xi_j\right) + \beta V\left(A', K', f'\right)$$

- quasi-linearity of preferences
  1. planner may wish to give more than one car to some, none to others
  2. precludes positive relation between c_j and newer (low i) vintages

- competitive equilibrium and interest rate sensitivity
  1. set of household assets will be important in determining sensitivity of $\mathbf{z}_j$ to changes in interest rates
  2. If ϕ is time-invariant, may be hard to get large e response to movements in A / interest rates

Durable Goods Example

How does the distribution evolve?

- ψ defines the law of motion where $f' = \psi(f, e)$
- $f = \{f_1, \ldots, f_i, \ldots\}$ where f_i is number of cars of vintage i
- $f_i - h_i$: number of vintage i cars scrapped.

$$
\begin{aligned}
h_i &= \sum_{j=1}^{J} z_{i,j} \text{ with } h_i \leq f_i \\
f'_{i+1} &= h_i \text{ for } i = 1, 2.... \\
f'_1 &= e
\end{aligned}
$$

- Note: Distribution of vintages *across households* not in the state vector, only the distribution of vintages itself.

Allocation of Capital across Plants

Environment

- $y = z\varepsilon k^{\alpha} n^{\nu}$ plant production

  z : aggregate shock, $\Pr\{z' = z_j \mid z = z_i\} = \pi_{ij}$

  ε : plant-specific shock, $\Pr\{\varepsilon' = \varepsilon_m \mid \varepsilon = \varepsilon_l\} = \pi^{\varepsilon}_{lm}$

  k and n: plant capital and labor

- Capital allocated to plants one period in advance: dynamic assignment

- No adjustment costs or indivisibilities $\Longrightarrow$ effective separability between assignment problem and aggregate accumulation problem

Allocation of Capital across Plants

Competitive equilibrium

- Competitive equilibrium allocations are simpler to characterize

$$V\left(\varepsilon_l, k; z_i, f\right) = \max_{n,k'}\Big[z_i \varepsilon_l k^{\alpha} n^{\nu} - \omega n - \left(k' - (1-\delta)k\right) + \sum_{j=1}^{N_z} \pi_{ij} d_j \sum_{m=1}^{N_\varepsilon} \pi^{\varepsilon}_{lm} V\left(\varepsilon_m, k'; z_j, f'\right)\Big]$$

- $\omega = \omega\left(z_i, f\right)$ equilibrium real wage
- $d_j = d_j\left(z_i, f\right)$ stochastic discount factor (household's marginal rate of substitution)

Allocation of Capital across Plants

Capital assignment characterized

- static allocation of labor: $\nu z \varepsilon k^{\alpha} n^{\nu-1} = \omega$

- efficiency condition for k' becomes:

$$1 - \sum_{j=1}^{N_z} \pi_{ij} d_j (1-\delta) = \alpha \nu^{\frac{\nu}{1-\nu}} \left(k'\right)^{\frac{\alpha+\nu-1}{1-\nu}} \sum_{j=1}^{N_z} \pi_{ij} d_j \left(\frac{z_j}{\omega_j^{\nu}}\right)^{\frac{1}{1-\nu}} \left[\sum_{m=1}^{N_\varepsilon} \pi^{\varepsilon}_{lm} \varepsilon_m^{\frac{1}{1-\nu}}\right]$$

- plant-level state vector is (ε_l, k), but only ε_l matters in determining k'

- plant-specific terms multiplicatively separable from aggregate terms

  $\Longrightarrow$ *shares* of capital allocated independently of aggregate state

Allocation of Capital across Plants

Time-invariant capital assignment rule

- let h_m be the time-invariant measure of plants with ε_m

- define $A(\varepsilon_l) = \left(\sum_{m=1}^{N_\varepsilon} \pi_{lm}^{\varepsilon} \varepsilon_m^{\frac{1}{1-\nu}} \right)^{\frac{1-\nu}{1-(\alpha+\nu)}}$

- capital of a plant that had ε_l last period is $k_l = \chi_l K$, where

$$\chi_l = \frac{A(\varepsilon_l)}{\sum_{m=1}^{N_\varepsilon} h_m A(\varepsilon_m)}, \ l = 1, \ldots, N_\varepsilon$$

Allocation of Capital and Labor across Plants

Costs of dynamic assignment

- Equilibrium is efficient, but how costly is dynamic allocation?

- What if we could assign current capital *after* seeing productivities?

  assuming idiosyncratic shock is 8 times as variable as aggregate (Cooper & Haltiwanger 2006), and remaining parameters taken from Khan & Thomas (2006) [$\alpha = 0.2565$, $\nu = 0.64$]...

  - *Endogenous* TFP would be 3.2 percent higher
  - Consumption and output 4.3 percent higher

- Suggests dynamic assignment may have large welfare implications in more realistic settings involving allocative distortions (e.g., capital adjustment costs, firing taxes).