Technical Appendix for 'Inventories and the business cycle: An equilibrium analysis of (S,s) policies'

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This technical appendix provides a more detailed description of the model and the solution method than is in the paper. For accessibility, wherever possible, we have organized results using section and equation numbers corresponding to those in the paper.

In section 2.2, we provide a more involved description of recursive competitive equilibrium, including a description of the stock market. As the representative household holds the market portfolio each period, there is no net trade in shares of firms, and the description of the stock market was omitted from the paper for simplicity and without loss of generality. For readability, this section is largely self-contained. In section 2.3, we include several results on intermediate and final good firms that are used in the numerical solution of the model. As above, we have repeated some of the text from the paper for completeness. Section 3.3 includes a complete description of the numerical method. There, we also present results on the accuracy of the model solution method.

2.2 Competitive equilibrium with a stock market

The description of the problem of a final goods firm, and the problem of the intermediate goods firm, is essentially unchanged from the paper. The household's problem is restated to allow for the buying and selling of shares in firms.

Recall that the model yields an endogenous distribution of final goods firms over inventory levels, $\mu : \mathcal{B}(\mathbf{S}) \rightarrow [0,1]$. The economy's aggregate state is (z, A), where $A \equiv (K, \mu)$ represents the endogenous state vector, K is the aggregate capital stock held by intermediate goods firms, and z is their total factor productivity. The distribution of final goods firms over inventory levels evolves according to $\mu' = \Gamma_\mu (z, A)$, and capital evolves according to $K' = \Gamma_K (z, A)$. Below, we summarize the law of motion governing the endogenous aggregate state by $A' = \Gamma(z, A)$.

The final good is the numeraire, and the aggregate state vector determines all equilibrium relative prices. Firms employ labor at real wage $\omega (z, A)$, and intermediate goods are traded at relative price $q (z, A)$. Next, $Q_j (z, A)$ is the price of an Arrow security that will deliver one unit of the final good next period if $z' = z_j$. Equilibrium in the asset markets requires that all firms discount future earnings using these state-contingent prices. For ease of exposition, we suppress the arguments of all price functions in listing the problems of firms.

Problem of a final goods firm: Let $v^0(s,\xi;z,A)$ represent the expected discounted value of a final goods firm with current inventory stock, s, and fixed cost draw, ξ, given the aggregate state (z,A). Recall that any such firm chooses whether or not to undertake active inventory adjustment prior to production, and, contingent on that decision, the firm selects its order for intermediate goods, $x_m \neq 0$. The details of this choice are reviewed below.

order size	total order costs	production-time stock	next-period stock
$x_m \neq 0$	$\omega\xi + qx_m$	$s_1 = s + x_m$	$s' = s_1 - m$
$x_m = 0$	0	$s_1 = s$	$s' = s_1 - m$

Next, given its stock of intermediate goods available for production, s_1, the firm chooses its employment $n \geq 0$, and future inventories, $s' \geq 0$. This determines its production net of storage costs, $y = G(s + x_m - s', n) - \sigma s'$. We state the problem facing such a firm recursively using equations (1) - (3),

$$v^0(s,\xi;z,A) = \max\left\{-\omega\xi + \max_{x_m \geq -s}[-qx_m + v^1(s+x_m;z,A)], v^1(s;z,A)\right\}, \tag{1}$$

where $v^1(s_1;z,A)$ summarizes the firm's expected discounted profits gross of current order costs, conditional on its available stock of intermediate goods at production time:

$$v^1(s_1;z,A) = \max_{n\geq 0,\ s'\geq 0}\Big[G(s_1 - s', n) - \sigma s' - \omega n + \sum_{j=1}^{N_z} Q_j v(s';z_j,A')\Big], \tag{2}$$

given the aggregate law of motion $A' = \Gamma(z,A)$. Finally, in equation (2), $v(s';z_j,A')$ represents the expected continuation value associated with future inventories s' if the aggregate state next period is (z_j, A'). This is the expectation of the firm's value taken over the adjustment cost:

$$v(s;z,A) = \int_{\underline{\xi}}^{\overline{\xi}} v^0(s,\xi;z,A)\,H(d\xi). \tag{3}$$

Intermediate goods firm's problem: Given its pre-determined capital stock, k, and the current aggregate state, (z,A), the representative intermediate goods firm chooses current employment, l, and its capital for the next period, k'. Its value, $w(k,z,A)$, solves the functional equation below:

$$w(k,z,A) = \max_{l\geq 0, k'\geq 0}\left(qzF(k,l) - \omega l - \big(k' - (1-\delta)k\big) + \sum_{j=1}^{N_z} Q_j w\left(k', z_j, A'\right)\right), \tag{4}$$

given $A' = \Gamma(z,A)$.

Household's problem: The representative household owns all firms in the economy and, at the beginning of each period, holds three types of assets. These include Arrow securities, a, and shares in final and intermediate good firms, $\widehat{\mu}_f(s)$ and $\widehat{\mu}_i$, respectively. Dividends from final goods firms with inventory holdings s are $D_f(s;z,A)$, and the ex-dividend price is $Q_f^1(s;z,A)$. The intermediate goods producer pays dividends $D_i(z,A)$, and its ex-dividend price is $Q_i^1(z,A)$. Finally, given total hours worked, n^h, household labor income is $\omega(z,A)\,n^h$.

The sum of the value of initial wealth plus labor income is allocated across current consumption, c, and purchases of new securities. The latter include purchases of Arrow securities, $\left(a_j'\right)_{j=1}^{N_z}$, and end-of-period shares in final goods firms, $\widehat{\mu}_f'(s')$, $s' \in \mathbf{S}$, and intermediate goods firms, $\widehat{\mu}_i'$. The price of a share in a final goods firm with end of period inventory holdings s' is $Q_f^0(s';z,A)$. Similarly, the price of a share in the representative intermediate goods firm is $Q_i^0(z,A)$.

The household chooses $\left(c, n^h, \left(a_j'\right)_{j=1}^{N_z}, \widehat{\mu}_f', \widehat{\mu}_i\right)$ to solve the problem below.

$$\begin{aligned} h\left(a,\widehat{\mu}_f,\widehat{\mu}_i;z,A\right) &= \max\left(u\left(c,1-n^h\right)+\beta\sum_{j=1}^{N_z}\pi_{ij}h\left(a_j',\widehat{\mu}_f',\widehat{\mu}_i';z_j,A'\right)\right) \qquad (5)\\ &\text{subject to} \end{aligned}$$

$$\begin{aligned} &a+\int_{\mathbf{S}}\left[D_f(s;z,A)+Q_f^1(s;z,A)\right]\widehat{\mu}_f(ds)+\left[D_i(z,A)+Q_i^1(z_j,A)\right]\widehat{\mu}_i\\ &+\omega(z,A)n^h \geq c+\sum_{j=1}^{N_z}Q_j(z,A)a_j'+\int_{\mathbf{S}}Q_f^0\left(s';z,A\right)\widehat{\mu}_f'\left(ds'\right)+Q_i^0(z,A)\,\widehat{\mu}_i' \end{aligned}$$

Additional constraints on the households purchases of securities are $a_j' \geq \underline{a}$, $j=1,\ldots,N_z$; $\widehat{\mu}_f'(s') \geq \underline{a}$ for all $s' \in \mathbf{S}$; and $\widehat{\mu}_i' \geq \underline{a}$ where $\underline{a}<0$. These constraints, which prevent Ponzi schemes, do not bind in equilibrium. Finally, the household also takes as given the evolution of the endogenous aggregate state, $A' = \Gamma(z,A)$.

Equilibrium: A *Recursive competitive equilibrium* is a set of functions, $(v, x_m, n, s', w, l, k', h, c, n^h, (a_j)_{j=1}^{N_z}, \widehat{\mu}_f', \widehat{\mu}_i', \omega, q, (Q_j)_{j=1}^{N_z}, Q_f^0, Q_i^0, Q_f^1, Q_i^1, D_f, D_i, \Gamma_K, \Gamma_\mu)$, satisfying the following.[1]

1. Firm and household decisions are optimal: (a) $x_m(s,\xi;z,A)$, $n(s,\xi;z,A)$ and $s'(s,\xi;z,A)$ solve the problem in (1) - (3), and $v(s;z,A)$ is the associated value function; (b) $l(k;z,A)$ and $k'(k;z,A)$ solve the problem in (4), and $w(k;z,A)$ is the associated value function; (c) $c\left(a,\widehat{\mu}_f,\widehat{\mu}_i;z,A\right)$, $n^h\left(a,\widehat{\mu}_f,\widehat{\mu}_i;z,A\right)$, $\left(a_j'\left(a,\widehat{\mu}_f,\widehat{\mu}_i;z,A\right)\right)_{j=1}^{N_z}$, $\widehat{\mu}_f'\left(s',a,\widehat{\mu}_f,\widehat{\mu}_i;z,A\right)$ where $s' \in \mathbf{S}$, and $\widehat{\mu}_i'\left(a,\widehat{\mu}_f,\widehat{\mu}_i;z,A\right)$ solve the problem in (5), and $h\left(a,\widehat{\mu}_f,\widehat{\mu}_i;z,A\right)$ is the associated value function.

2. Markets for final goods, intermediate goods, labor and assets clear:

(a) $c(0,\mu,1;z,A)+k'(K;z,A)-(1-\delta)K = \int_{\mathbf{S}\times[\underline{\xi},\overline{\xi}]} y(s,\xi;z,A)\,H(d\xi)\,\mu(ds)$,
where $y(s,\xi;z,A) = G\Big(s+x_m(s,\xi;z,A)-s'(s,\xi;z,A),\,n(s,\xi;z,A)\Big)-\sigma s'(s,\xi;z,A)$;

(b) $\int_{\mathbf{S}\times[\underline{\xi},\overline{\xi}]} x_m(s,\xi;z,A)\,H(d\xi)\,\mu(ds) = zF\Big(K, l(K;z,A)\Big)$;

(c) $n^h(0,\mu,1;z,A) = l(K;z,A)+\int_{\mathbf{S}\times[\underline{\xi},\overline{\xi}]}\left[n(s,\xi;z,A)+\chi\left(x_m(s,\xi;z,A)\right)\xi\right]H(d\xi)\,\mu(ds)$,
where $\chi(x_m)=1$ for $x_m \neq 0$.

[1] To avoid additional notation, we use choice variables to denote decision rules.

(d) $\widehat{\mu}'_f(s,0,\mu,1;z,A)=\mu'(s)$ for all $s\in\mathbf{S}$, $\widehat{\mu}'_i(0,\mu,1;z,A)=1$, and $a'_j(0,\mu,1;z,A)=0$ for $j=1,\ldots,N_z$.

3. Laws of motion for aggregate state variables are consistent with individual decisions:
(a) $\mu'\left(\widetilde{S}\right)=\int_{\left\{(s,\xi)|s'(s,\xi;z,A)\in\widetilde{S}\right\}}H(d\xi)\,\mu(ds)$ for all $\widetilde{S}\in\mathcal{B}(\mathbf{S})$ defines $\Gamma_\mu(z,A)$;
(b) $K'=k'(K;z,A)$ defines $\Gamma_K(z,A)$.

Several comments are useful here. First, the equilibrium prices at which households purchase shares in firms are

$$\begin{aligned} Q_f^0(s;z,A) &= \sum_{j=1}^{N_z} Q_j(z,A)\, v\Big(s;z_j,\Gamma(z,A)\Big) \\ Q_i^0(z,A) &= \sum_{j=1}^{N_z} Q_j(z,A)\, w\Big(k'(K;z,A);z_j,\Gamma(z,A)\Big). \end{aligned}$$

The household is able to diversify the idiosyncratic risk faced by final goods firms, ξ, as it holds a large number of firms of each type s. Thus, the dividends and ex-dividend prices it receives for its shares in these firms are

$$\begin{aligned} D_f(s;z,A) &= \int_{\underline{\xi}}^{\overline{\xi}} [y(s,\xi;z,A)-q(z,A)\,x_m(s,\xi;z,A)-\omega(z,A)\,n(s,\xi;z,A)]H(d\xi) \\ Q_f^1(s;z,A) &= \int_{\underline{\xi}}^{\overline{\xi}} \sum_{j=1}^{N_z} Q_j(z,A)\, v^0\Big(s'(s,\xi;z,A),z_j,\Gamma(z,A)\Big)H(d\xi). \end{aligned}$$

Similar expressions hold for shares in the intermediate goods firm:

$$\begin{aligned} D_i(z,A) &= q(z,A)\,zF\Big(K,l(K;z,A)\Big)-\omega(z,A)\,l(K;z,A)-[k'(K;z,A)-(1-\delta)\,k] \\ Q_i^1(z,A) &= \sum_{j=1}^{N_z} Q_j(z,A)\, w\left(k'(K;z,A);z_j,\Gamma(z,A)\right). \end{aligned}$$

Finally, the representative household holds the distribution of firms in equilibrium, and there is no net supply of Arrow securities; $a=0$, $\widehat{\mu}_f=\mu$, and $\widehat{\mu}_i=1$. As a result, household consumption and total hours worked may be written simply as functions of the aggregate state, $C(z,A)$ and $N(z,A)$.

2.3 Firm behavior and inventory adjustment

In this section, we derive a restriction on the equilibrium price, $p(z,A)$ using the intermediate goods firm's problem. Next we restate the problem faced by final goods firms for convenience, so that we may refer to it below. Finally, we use the characterization of the inventory adjustment policy of final goods firms to simplify the law of motion of the distribution of such firms over inventory levels. These results are all used in the description of the numerical method in section 3.3.

Our numerical solution exploits a homogeneity property of the intermediate good producer's value function that is derived here. Suppressing the arguments of p, q and ω, recall that the value function W solves

$$W(k; z, A) = \max_{k', l} \left(p \left[qzF(k, l) + (1 - \delta) k - k' - \omega l \right] + \beta \sum_{j=1}^{N_z} \pi_{ij} W\left(k'; z_j, A'\right) \right). \tag{6}$$

The following efficiency conditions describe the producer's selection of employment and investment:

$$zD_2 F(k, l) = \frac{\omega}{q} \tag{A1}$$

$$\beta \sum_{j=1}^{N_z} \pi_{ij} D_1 W\left(k'; z_j, A'\right) = p. \tag{A2}$$

Having assumed that F is linearly homogenous, the producer's decision rules for employment and production are proportional to its capital stock; $l(k; z, A) \equiv L(z, A)k$, where $L(z, A)$ solves (A1) given ω and q, and $x(k; z, A) = zF(1, L(z, A))k$. This means that current profits, $\pi(z, A)k$, are linear in k, as is the firm's value function, $W(k; z, A) = \widehat{w}(z; A) k$. Equation (A2) then implies that an interior choice of investment places the following restriction on the equilibrium price of final output:

$$p(z, A) = \beta \sum_{j=1}^{N_z} \pi_{ij} \widehat{w}\left(z_j, A'\right). \tag{A3}$$

When (A3) is satisfied, the intermediate goods firm is indifferent to any level of k' and purchases investment equal to the final goods remaining after households' consumption.

We now turn to final goods firms. For completeness, we reiterate several definitions from the paper in equations (7) - (10). First, recall that $V^0(s, \xi; z, A)$ represents the reformulated expected discounted value of a final goods firm with start-of-date inventory holdings s and fixed order cost ξ. The beginning of period expected value of the firm prior to the realization of its fixed cost is

$$V(s; z, A) \equiv \int_{\underline{\xi}}^{\overline{\xi}} V^0(s, \xi; z, A) H(d\xi). \tag{7}$$

Next, $V^1(s_1; z, A)$ represents the value of entering production with inventories s_1. Given this stock available for production, the firm selects its current employment, its inventories for next period, and hence the amount of its stock used in current production, to solve:

$$V^1(s_1; z, A) = \max_{s' \geq 0, n \geq 0} \left(p \left[G\left(s_1 - s', n\right) - \omega n - \sigma s' \right] + \beta \sum_{j=1}^{N_z} \pi_{ij} V\left(s'; z_j, A'\right) \right). \tag{8}$$

Given the continuation value of inventories, $V(s'; z_j, A')$, equation (8) yields both the firm's employment (in production) decision and its use of intermediate goods. Let $N(s_1; z, A)$ describe its employment and $S(s_1; z, A)$ its stock of intermediate goods retained for future use. Its net production of final goods is then $Y(s_1; z, A) = G(s_1 - S(s_1; z, A), N(s_1; z, A)) - \sigma S(s_1; z, A)$. Thus, we have decision rules for employment, production, and next-period inventories as functions of the production-time stock s_1.

Given the middle-of-period valuation of the firm, V^1, we now examine the inventory adjustment decision made by a final goods firm entering the period with inventories s and drawing adjustment cost ξ. Equations (9) - (10) describe the firm's determination of (i) whether to place an order and (ii) the target inventory level with which to begin the production sub-period, conditional on an order.

$$\begin{aligned} V^0(s,\xi;z,A) &= pqs + \max\Big\{-p\omega\xi + V^a(z,A), -pqs + V^1(s;z,A)\Big\} && (9) \\ V^a(z,A) &\equiv \max_{s_1 \geq 0}\Big(-pqs_1 + V^1(s_1;z,A)\Big). && (10) \end{aligned}$$

Given the derivation of the target level of intermediate goods, $s^*(z,A)$, the associated value of adjustment, $V^a(z,A)$, and the threshold adjustment cost, $\xi^T(s;z,A)$, we can rewrite the beginning of period expected value of a final good firm prior to the realization of its fixed delivery cost as

$$\begin{aligned} V(s;z,A) = pqs + H\left(\xi^T(s;z,A)\right)V^a(z,A) - p\omega\int_{\underline{\xi}}^{\xi^T(s;z,A)}\xi H(d\xi) \qquad (A4) \\ +\Big(1 - H\left(\xi^T(s;z,A)\right)\Big)\Big(V^1(s;z,A) - pqs\Big), \end{aligned}$$

where $\int_{\underline{\xi}}^{\xi^T(s;z,A)}\xi H(d\xi)$ is the conditional expectation of the fixed cost ξ.

Finally, we examine Γ_μ, the evolution of the distribution of final goods firms. Of each group of firms sharing a common stock $s \neq s^*$ at the start of the current period, fraction $1 - H(\xi^T(s;z,A))$ do not adjust their inventories. Thus, with some abuse of notation, $\mu(s)[1 - H\Big(\xi^T(s;z,A)\Big)]$ firms will begin the next period with $S(s;z,A)$ as defined above. Those firms that either enter the period with the current target or actively adjust to it for production, $\mu(s^*(z,A)) + \int_{\mathbf{S}} H\Big(\xi^T(s;z,A)\Big)\mu(ds)$ in all, will move to the next period with $S(s^*(z,A);z,A)$.

Given the preceding discussion, the evolution of the distribution of final goods firms may be described as follows. Define $S^{-1}(\widetilde{s};z,A)$ as the production-time inventory level that gives rise to next period inventories $\widetilde{s}$ in the solution to (8). For any stock $\widetilde{s}$ other than that arising from the target level of production-time inventories, $S^{-1}(\widetilde{s};z,A) \neq s^*(z,A)$,

$$\mu'(\widetilde{s}) = \Big[1 - H\Big(\xi^T\left(S^{-1}(\widetilde{s};z,A)\right)\Big)\Big]\mu\Big(S^{-1}(\widetilde{s};z,A)\Big). \qquad (A5)$$

For the stock arising from the target inventory level, $S^{-1}(\widetilde{s};z,A) = s^*(z,A)$,

$$\mu'(\widetilde{s}) = \mu\Big(s^*(z,A)\Big) + \int_{\mathbf{S}} H\Big(\xi^T(s;z,A)\Big)\mu(ds). \qquad (A6)$$

3.3 Numerical method

The solution algorithm is iterative, applying one set of forecasting rules to generate decision rules that are used in obtaining data upon which to base the next set of forecasting rules. In particular, given I, we assume functional forms that predict next period's endogenous state (K', m'), and the prices p and pq, as functions of the current state, $K' = \widehat{\Gamma}_K\left(z, K, m; \chi_l^K\right)$, $m' = \widehat{\Gamma}_m\left(z, K, m; \chi_l^m\right)$, $p = \widehat{p}\left(z, K, m; \chi_l^p\right)$ and $pq = \widehat{pq}\left(z, K, m; \chi_l^{pq}\right)$, where χ_l^K, χ_l^m, χ_l^p, and χ_l^{pq} are parameter vectors that are determined iteratively, with l indexing these iterations. For the class of utility functions we use, the wage is immediate once p is specified; hence there is no need to assume a wage forecasting function.

For any I, $\widehat{\Gamma}_K$, $\widehat{\Gamma}_m$, $\widehat{p}$, and $\widehat{pq}$, we solve for V on a grid of values for $(s; z, K, m)$. Next, we simulate the economy for T periods, recording the actual distribution of final goods firms, μ_t, at the start of each period, $t = 1, \ldots, T$. To determine equilibrium at each date, we begin by calculating m_t using the actual distribution, μ_t, and then we use $\widehat{\Gamma}_K$ and $\widehat{\Gamma}_m$ to specify expectations of K_{t+1} and m_{t+1}. This determines $\beta \sum_{j=1}^{N_z} \pi_{ij} w\left(z_j, K_{t+1}, m_{t+1}\right)$ and $\beta \sum_{j=1}^{N_z} \pi_{ij} V\left(s'; z_j, K_{t+1}, m_{t+1}\right)$ for any s'. Given the second function, the conditional expected continuation value associated with any level of inventories, we can determine $s^*\left(z, K, m\right)$ and $\xi^T\left(s; K, m\right)$, hence recovering the decisions of final goods firms and thus next period's distribution, for any values of p and q. Given any p, the equilibrium q is solved to equate the supply of intermediate goods, $x(K; z, A)$, to the demand generated by final goods firms.[2] The equilibrium output price, $p(z, A; \chi_l^K, \chi_l^m, \chi_l^p, \chi_l^{pq})$, is that which generates production of the final good such that, given $c = \frac{1}{p}$, the residual investment implies a level of future capital, $K_{t+1} = (1-\delta) K_t + Y_t - c_t$, that satisfies the restriction in (A3). Finally, (A5) and (A6) determine the distribution of final goods firms over inventory levels for next period, μ_{t+1}. With the equilibrium K_{t+1} and μ_{t+1}, we move to the next date in the simulation, again solving for equilibrium, and so forth. Once the simulation is completed, the resulting data, $(p_t, p_t q_t, K_t, m_t)_{t=1}^T$, are used to re-estimate $\left(\chi_l^K, \chi_l^m, \chi_l^p, \chi_l^{pq}\right)$ using OLS.

We repeat this two-step process, first solving for V given $\left(\chi_l^K, \chi_l^m, \chi_l^p, \chi_l^{pq}\right)$, next using our solution for firms' value functions to determine equilibrium decisions over a simulation, storing the equilibrium results for $(p_t, p_t q_t, K_t, m_t)_{t=1}^T$, and then updating $\left(\chi_{l+1}^K, \chi_{l+1}^m, \chi_{l+1}^p, \chi_{l+1}^{pq}\right)$, until these parameters converge. The number of partition means used to proxy for the distribution μ, I, is chosen such that agents' forecasting rules are sufficiently accurate.

Table B1 displays the actual forecasting functions used in the baseline inventory model, based on a $10,000$ period simulation. We use a log-linear functional form for each forecasting rule that is conditional on the level of productivity, z_i, $i = 1, \ldots, N_z$.[3] In the results reported here, $I = 1$. This means that, alongside z and K, only the mean of the current distribution of firms over inventory levels, start-of-period aggregate inventory holdings, is used by agents to forecast the relevant features of the future endogenous state. This degree of approximation would be unacceptable if it yielded large errors in forecasts. However, table B1 shows that, for each of the two values of productivity, the

[2] This demand depends on the target inventory level $s^*\left(z, K, m\right)$, the start-of-period distribution of firms $\mu(s)$, and the adjustment thresholds of each firm type, $\xi^T\left(s; K, m\right)$, as seen in equation (16) of the paper.

[3] We have tried a variety of alternatives, including adding higher-order terms and a covariance term. None of these significantly altered the forecasts used in the model.

forecast rules for prices and both elements of the approximate state vector are extremely accurate. The standard errors across all regressions are small, and the R^2's are high, all above 0.999.

The regressions in table B1 also offer some insight into the impact of inventories on the model, as they provide a description of the behavior of equilibrium prices and the laws of motion for capital and inventories. In particular, note that there is relatively little impact of inventories, m_1, on the valuation of current output, p, and on capital, K. Inventories have somewhat larger influence in determining the price of intermediate goods and, of course, their own future level.

References

[1] Krusell, P. and A. A. Smith Jr. (1997) 'Income and wealth heterogeneity, portfolio choice, and equilibrium asset returns' *Macroeconomic Dynamics* 1, 387-422.

[2] Krusell, P. and A. A. Smith Jr. (1998) 'Income and wealth heterogeneity in the macroeconomy' *Journal of Political Economy* 106, 867-896.

Table B1: Forecasting rules in the baseline inventory economy*

		z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	z_9
	obs.	420	836	1348	1646	1734	1646	1207	746	417
pq	β_0	0.596	0.587	0.572	0.560	0.544	0.523	0.498	0.479	0.461
	β_1	-0.473	-0.479	-0.480	-0.486	-0.487	-0.485	-0.478	-0.476	-0.473
	β_2	-0.106	-0.010	-0.098	-0.091	-0.088	-0.090	-0.095	-0.095	-0.095
	S.E.	2.6×10^{-4}	2.9×10^{-4}	3.0×10^{-4}	2.9×10^{-4}	2.9×10^{-4}	2.9×10^{-4}	3.0×10^{-4}	2.9×10^{-4}	2.6×10^{-4}
	adj. R^2	0.9997	0.9997	0.9997	0.9998	0.9997	0.9998	0.9997	0.9997	0.9997
p	β_0	1.370	1.367	1.362	1.357	1.351	1.343	1.334	1.325	1.320
	β_1	-0.332	-0.333	-0.334	-0.335	-0.334	-0.333	-0.330	-0.326	-0.326
	β_2	-0.050	-0.048	-0.046	-0.044	-0.044	-0.043	-0.044	-0.045	-0.044
	S.E.	0.1×10^{-4}	0.1×10^{-4}	0.2×10^{-4}	0.2×10^{-4}	0.3×10^{-4}	0.3×10^{-4}	0.3×10^{-4}	0.3×10^{-4}	0.2×10^{-4}
	adj. R^2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
K'	β_0	0.077	0.073	0.072	0.070	0.072	0.074	0.085	0.093	0.091
	β_1	0.880	0.885	0.888	0.891	0.892	0.892	0.883	0.876	0.882
	β_2	0.029	0.023	0.020	0.016	0.016	0.015	0.021	0.024	0.021
	S.E.	1.3×10^{-4}	2.9×10^{-4}	3.0×10^{-4}	2.9×10^{-4}	2.9×10^{-4}	2.9×10^{-4}	3.0×10^{-4}	2.9×10^{-4}	2.6×10^{-4}
	adj. R^2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	1.0000
m_1'	β_0	-0.318	-0.296	-0.275	-0.246	-0.239	-0.237	-0.258	-0.274	-0.263
	β_1	0.220	0.209	0.195	0.174	0.172	0.176	0.204	0.229	0.225
	β_2	0.702	0.710	0.722	0.741	0.744	0.741	0.721	0.707	0.704
	S.E.	6.3×10^{-4}	6.8×10^{-4}	7.0×10^{-4}	6.7×10^{-4}	6.9×10^{-4}	6.9×10^{-4}	7.0×10^{-4}	6.7×10^{-4}	5.9×10^{-4}
	adj. R^2	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996

*Forecasting rules conditional on current productivity: $\log(X) = \beta_0 + \beta_1[\log(K)] + \beta_2[\log(m_1)]$, with X = pq, p, K', and m_1'.