Appendix for 'Inventories and the business cycle: An equilibrium analysis of (S,s) policies'

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This appendix provides additional results for the paper. For accessibility, where appropriate, we have organized results using section and equation numbers corresponding to those in the paper. In section 9 we discuss several additional exercises which test the robustness of the conclusions drawn from our basic (S,s) model of inventories. In 9.1 we explore different placement for a single technology shock, and in 9.2 we study preference shocks. Next, in 9.3 we report the effects of changing aggregate returns to scale. A reduced form model, where inventories are a factor of production, is described in section A. In A.1 we explore an extension of this model where capital is used both in the production of intermediate and final goods.

9 Additional robustness results

In our paper, we arrived at two results that contradict earlier findings about the cyclical implications of inventories. First, in our model, inventory accumulation does not amplify aggregate fluctuations, nor do smaller average stocks imply reduced GDP volatility. Second, the model generates a countercyclical inventory-to-sales ratio when aggregate fluctuations are driven by technology shocks, despite the assumption of perfect competition. In this section, we explore the robustness of these two results to model specification and parameter values.

In sections 9.1 - 9.2, we consider alternative sources of aggregate fluctuations, first modifying the incidence of technology shocks across firms and, second, instead introducing preference shocks. In each case, we find that the rise in GDP volatility associated with the presence of inventories remains small, and the inventory-sales ratio remains countercyclical. However, in contrast to our baseline formulation of the model, each of these alternative specifications performs poorly with regard to the inventory facts. In section 9.3, we return to our baseline formulation of the model and examine the effects of raising aggregate returns to scale in three distinct ways. In each case, we find that the differences between the model with inventories and the corresponding control model narrow relative to our calibrated comparison in section 6.2 of the paper. Interestingly, in two of these cases, the inclusion of inventories is seen to marginally reduce the cyclical volatility of GDP.[1]

[1] The discussion throughout this section is based on the second moment tables provided, as well as impulse response

9.1 Technology shock location

In the baseline formulation of our model, we assumed that the business cycle originates from technology shocks that directly affect only the production of intermediate goods. Given that our inventories are stocks of intermediate goods, this assumption was designed to yield consistency with the countercyclical relative price of inventories in the data. Here, we consider alternative formulations that do not share this consistency, one where shocks affect only production of final goods and one where shocks evenly affect all firms in the economy. Results for these cases are presented in panels A and B of table A1. In each case, we use the method described in section 3.1 to re-estimate the shock process and discretise the result as a two-state Markov Chain; other parameters are maintained at their previous values.

Final goods shock: When shocks directly affect only final goods production, the relative price of intermediate goods (inventories) is strongly procyclical. Here, a positive shock raises the productivity of labor and intermediate goods in the final goods sector, increasing demand for intermediate goods. With no change in productivity in the intermediate goods sector, however, the supply of these goods rises only gradually as capital is accumulated. In the control model, although final goods firms respond by shifting to more labor-intensive production, their output rises slowly, given diminishing marginal productivity of labor.

When the same shock hits the inventory model, increased demand for intermediate goods in production is satisfied not only by increased production, but also by decummulation of existing stocks. An episode of negative inventory investment delivers a sharp rise in the use of intermediate goods, and hence employment, in final production. Given the sector's high productivity, the resulting spike in final sales far outweighs the negative inventory investment; over this episode, the rise in GDP exceeds that in the control model. This explains the 5.4 basis point rise in GDP volatility relative to the control, which, while small, is roughly double that in our baseline results.

In contrast to our baseline inventory model, this reformulation is inconsistent with the cyclical behavior of inventories. Initially, the sharp rise in final sales and GDP coincides with negative inventory accumulation; thereafter, once stocks are sufficiently low, inventories are replenished at the expense of production. Thus, inventory investment is negatively correlated with both sales and GDP, and sales volatility exceeds that of production. Finally, the model's countercyclical inventory-sales ratio arises immediately from the countercyclicality of inventory investment.

Economywide shock: In our baseline inventory model, a positive shock to the intermediate goods sector generated a greater rise in GDP relative to the control model through the additional demand for intermediate goods by firms seeking to increase inventories. Conversely, a positive shock to final goods production delivered a greater GDP rise as final goods firms drew down their stocks of intermediate goods to raise sales. The economywide shock model, where a positive shock evenly raises productivity in both sectors, lies between these two cases.

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figures that are available from the authors on request.

Because intermediate goods have diminishing marginal productivity when used in the production of final goods, ceteris paribus, a productivity shock to the intermediate goods sector raises final output by less than a same-sized shock to the final goods sector. As a result, the shock to final goods production plays the dominant role in the response to an economywide shock, and behavior here is essentially a muted version of the final goods shock case. As there, the relative price of intermediate goods is procyclical, and stocks are decummulated following a positive shock. However, this decummulation is minor, and it is quickly reversed as rapid capital accumulation boosts the rise in intermediate goods production. As a result, inventory investment is very weakly procyclical, has little volatility and is essentially uncorrelated with sales, while sales volatility roughly equals that of GDP. Thus, with respect to the inventory facts, this version of the model also performs poorly relative to our baseline inventory model.

9.2 Alternative shocks

We examine the behavior of our (S,s) inventory model when aggregate fluctuations are driven by shocks to preferences, rather than technology, in Khan and Thomas (2004). Here, we briefly summarize the results of that study, and report selected moments in panel C of table A1. While shocks to the household discount factor are a natural choice in a model abstracting from capital accumulation (such as Fisher and Hornstein (2000)), these shocks are problematic in a calibrated business cycle model with capital. Here, they generate extreme investment volatility and countercyclical consumption.[2] Thus, we instead examine shocks to households' marginal utility of consumption. These shocks affect not only the intertemporal tradeoff between consumption across dates, but also the intratemporal tradeoff between consumption and leisure, so they generate large procyclical labor supply responses, while capital accumulation takes on a lesser role. Specifically, we assume that period utility is $u(c, 1 - n^h) = z \log c + \eta \cdot (1 - n^h)$, and we estimate the parameters governing the preference shock, z, using the approach described in section 3.1, while maintaining all other parameters at their baseline values.

A persistent positive shock to preferences generates an urgency for current consumption and a decline in the relative valuation of leisure. Hours worked rise sharply, as do the outputs of each sector, while the relative price of intermediate goods is unchanged absent any changes in total factor productivity. With this shock, households are far more willing to sacrifice leisure than consumption, so only a small portion of the increased output is devoted to capital accumulation. Instead, the rise in GDP is propagated by persistently high labor supply.

In the model with inventories, urgency for consumption prompts final goods firms to draw upon their existing stocks to supplement their increased orders for intermediate goods. Thus, the response is similar to that following the technology shock to final goods firms in section **??**. As in that

[2] A shock to the discount factor increases both consumption and leisure. The resulting decline in hours worked reduces production, while sharp reductions in investment are used to allow increased consumption. Thus, consumption and leisure rise, while output and investment fall.

case, final sales and GDP initially rise more than they do in the corresponding control model, then decline somewhat as stocks are rebuilt. Here, however, there is less decummulation, as intermediate goods are made more productive by larger rises in employment. Consequently, inventory investment exhibits very little volatility, but its countercyclical movements nonetheless yield changes in final sales sufficient to imply a 9 basis point increase in GDP volatility over that in the model without inventories. (While this difference is small, it is nearly four times that in our baseline comparison.) Finally, when fluctuations arise from preference shocks, the negative comovement between sales and inventory investment necessarily generates countercyclical movements in the inventory-sales ratio, this model's only success with regard to the inventory facts.

9.3 Returns to scale

In this section, we return to the baseline formulation of our model and now consider the influence of returns-to-scale in its predictions. While our calibration of the model implies returns-to-scale in production at 0.827, there is little consensus about this value in the data. Thus, here we briefly examine how the results change when returns are raised to a value of 0.90 in each of three ways. First, we increase the share to labor in final goods production, θ_N, maintaining the average capital-output ratio at its baseline value, while allowing total labor share to rise. Second, we again increase θ_N, this time holding total labor share fixed and allowing the capital-output ratio to rise. Third, we raise the share to intermediate goods, θ_M, holding total labor share fixed, again allowing the capital-output ratio to rise. Results are reported in table A2, and labeled *High RTS Case 1, 2 and 3*, respectively. In each case, we adjust the remaining parameters to maintain all baseline calibration targets other than those explicitly noted, including the average inventory-sales ratio, and we re-estimate the shock process given the new parameter set.

In each of these cases of raised returns-to-scale, the qualitative response to a positive productivity shock is unchanged relative to our discussion of the baseline inventory economy in section 6.2. As there, the increase in production of intermediate goods exceeds the increase in their use, as aggregate inventories are gradually raised to reduce the frequency of orders and the adjustment costs entailed. Because supply of these goods is initially hindered by the scarcity of capital, this procyclical inventory accumulation dampens the rise in final sales, and, (relative to the corresponding model without inventories,) yields little or no additional rise in total production.

With higher returns to scale, dispersion in the distribution of production causes less output loss. As a result, orders and production are more concentrated among firms drawing low adjustment costs. This, in turn, makes inventory accumulation less important. Thus, relative to the baseline inventory model, there is less increase in inventory accumulation following a rise in productivity. Overall, responses are closer to their counterparts in the models without inventories, and differences in GDP volatility are even smaller than before. Interestingly, in the cases where increased returns coincide with a raised capital share (cases 2 and 3), GDP volatility marginally falls in the presence of inventories. In these cases, scarcity of capital has greater effect in slowing intermediate goods production. As a result, procyclical rises in inventory investment cause sufficient dampening in final sales as to reduce GDP volatility. Alongside the discussion in section 6.3, this may be taken as further evidence of the importance of capital in our study of the cyclical role of inventories.

A Reduced-form model

In this section, we develop a *basic reduced form model* where inventories are a factor of production with positive marginal product. It must be noted that such an exercise assumes rather than derives a role for inventories. To be clear, in our (S,s) model of inventories, firms may always set $s = 0$ and order intermediate goods every period. Instead, they choose to hold inventories in order to economize on the nonconvex costs of placing orders. Inventory accumulation is an optimal policy. By contrast, in the reduced form model we describe here, inventories are accumulated because they are an essential factor of production.

We present the results of this exercise for two reasons. First, in this reduced-form model, the implication of inventory investment for GDP volatility does not change with the location of technology shocks. Second, while this model is similar in spirit to that studied by Bils and Kahn (2000), it nonetheless generates a countercyclical inventory to sales ratio when the business cycle is driven by shocks to technology and all markets are perfectly competitive.

For comparability with our (S,s) model of inventories, the structure of the economy is similar except for the nature of inventories. In particular, intermediate goods are produced with capital and labor, and final goods are produced using the intermediate good and labor. The point of departure from our (S,s) model is that there are no costs of adjusting stocks of the intermediate good. Rather, firms must now accumulate two factors of production, capital and inventories, and investment in either requires final goods. Thus

$$K_{t+1} = (1-\delta) K_t + I_t \tag{A7}$$

$$S_{t+1} = (1-\delta_S) S_t + J_t \tag{A8}$$

where K_t is the capital stock at time t, I_t is investment in capital, S_t is the stock of inventories at time t, and J_t is gross inventory investment. The rate of depreciation of capital is δ, and the rate of depreciation of inventories is δ_S.

The production of final goods, Y_t, is allocated to consumption, C_t, capital investment and inventory investment.

$$Y_t \geq C_t + I_t + J_t \tag{A9}$$

Final goods are produced using inventories, intermediate goods, X_t, and labor, N_t:

$$Y_t = G(S_t, X_t, N_t). \tag{A10}$$

As there are no fixed costs of ordering intermediate goods, the use of intermediate goods equals their production, $M_t = X_t$, and we do not distinguish these series.

The production of intermediate goods is undertaken using capital and labor, L_t.

$$X_t = z_t F(K_t, L_t) \tag{A11}$$

Total factor productivity is a log-normal stochastic process with persistence ρ and a variance of innovations of σ_ε^2 as described in section 3.1 of the paper. In particular, $\log z_{t+1} = \rho \log z_t + \varepsilon_{t+1}$ with $\varepsilon_{t+1} \sim N(0, \sigma_\varepsilon^2)$.

Finally, the representative household values consumption and leisure each period.

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - N_t - L_t) \tag{A12}$$

The planner's problem is to maximize (A12) by choice of $\{C_t, N_t, L_t, I_t, J_t\}$ subject to (A7) - (A11), given (z_0, K_0, S_0) and the additional constraints that $C_t \geq 0$, $N_t \geq 0$, $L_t \geq 0$ and $1 - N_t - L_t \geq 0$.

In calibrating the model, we assume that $F(K, L)$ and $u(c, 1 - N - L)$ have the same functional form as in our (S,s) inventory model: $F(K, L) = K^\alpha L^{1-\alpha}$ and $u(C, 1 - N - L) = \log C + \eta(1 - N - L)$. The production of final goods now takes the form $G(S, M, N) = S^{\theta_s} M^{\theta_m} N^{\theta_n}$. We choose the same values for δ and β as in our (S,s) inventory model, which ensures the same investment-to-capital ratio and real interest rate in the steady state. Next $\delta_S = 0.0287$, so that the unit cost of storing inventories equals that in our model.

The capital-output ratio, labor's share of production and the average inventory to sales ratio are all set to the same values as in our (S,s) model (see sections 3.1 - 3.2 of the paper). Here, this requires that $\alpha = 0.372$, $\theta_n = 0.327$, and $\theta_S = 0.032$. Next, setting $\eta = 2.172$ ensures that hours worked are on average a third of total time. The stochastic process for z_t is identified by $(\rho, \sigma_\varepsilon^2)$; these parameter values are set to the values estimated using the control model described in the paper. Finally, we develop a no-inventory counterpart model by simply setting θ_S near 0 while maintaining all other parameter values.

We solve this model using standard linear methods. As seen in table A3, the reduced-form model behaves similarly to our (S,s) inventory model. Production is more variable than sales $(C_t + I_t)$, net inventory investment $(S_{t+1} - S_t)$ is procyclical, and, perhaps most importantly, the inventory to sales ratio is countercyclical.

In this model, inventories increase the variability of GDP by only 7 basis points. Note that, when we instead have technology shocks to the production of final goods, there is no change in this increase or in the cyclicality of inventory investment. (The results for this alternate location for the technology shock are available upon request.) Thus, in this reduced-form model, the location of the technology shock does not affect the role of inventory investment in increasing GDP volatility.

The countercyclicality of the inventory-to-sales ratio is a response to procyclical changes in real interest rates, which increase the cost of inventory investment. Such changes slow inventory accumulation and, as a result, final sales rises faster than the stock of inventories. This is exactly the reason why the capital-to-output ratio is countercyclical in a standard real business cycle model. Put differently, if we examined a partial equilibrium version of this model with a constant real interest rate, the inventory to sales ratio would be constant. This result suggests that the predictions of the model studied by Bils and Kahn (2000) hinge on their partial equilibrium analysis more than their reduced-form motive for inventories.

A.1 Reduced form model with capital in final good production

In section 5.3 of the paper, we explained how the slow accumulation of capital leads to a gradual increase in the production of intermediate goods which, in turn, implies that procyclical net inventory investment necessarily dampens changes in final sales. In our (S,s) model of inventories capital is used only in the production of intermediate goods. Holding constant the aggregate capital stock, if capital were used in the production of final goods, then this will reduce capital's share in the production of intermediate goods. Here we explore how this might change the central role of capital in our existing results using our reduced form model. We find that the aggregate capital to output ratio fully determines the effect of capital, and the fraction of capital in each sector is irrelevant to this effect.

To introduce capital as a factor of production in both sectors, we replace equations (A10) - (A11) the following,

$$Y_t = G\left(S_t, X_t, N_t, K_t^F\right) \tag{A11}$$

$$X_t = z_t F\left(K_t^I, L_t\right). \tag{A12}$$

The aggregate stock of capital, K_t, is allocated to the production of final goods, K_t^F, or intermediate goods, K_t^I,

$$K_t = K_t^F + K_t^I. \tag{1}$$

The reduced form inventory model with capital used in the production of final goods is now described by the solution of a planner's problem that maximizes is to maximize (A12) by choice of $\left\{C_t, N_t, L_t, K_t^F, K_t^I, I_t, J_t\right\}$ subject to (A7) - (A9) and (A11) - (1), given (z_0, K_0, S_0) and the additional constraints that $C_t \geq 0$, $N_t \geq 0$, $L_t \geq 0$ and $1 - N_t - L_t \geq 0$.

To explore the effect of introducing capital into the production of final goods, we solve two parameterizations of this model, labelled *low* k^F and *high* k^F. Assuming $G\left(S, X, N, K^F\right) = S^{\theta_s} M^{\theta_m} N^{\theta_n} \left(K^F\right)^{\theta_k}$, we set $\theta_k = 0.001$ in the *low* k^F case and $\theta_k = 0.165$ in the *high* k^F. Given the selection of this parameter, the remainder of the economy is calibrated exactly as described in the previous section. Specifically, we choose the same values for δ, δ_S, θ_m and β as before, and set α, θ_n and θ_s to match the aggregate capital-output ratio, labor's share of production and the average inventory to sales ratio in our (S,s) model. Next, we set η to ensure that, as before, hours worked is a third of time. In the *low* k^F case, $(\alpha, \theta_n, \theta_S, \eta) = (0.370, 0.326, 0.032, 2.172)$. For the *high* k^F case, $(\alpha, \theta_n, \theta_S, \eta) = (0.042, 0.162, 0.032, 2.172)$. We use the same stochastic process for z

as in our basic reduced form model discussed in section A, and, as before, we develop a no-inventory counterpart model by simply setting θ_S near 0 while maintaining all other parameter values.

As already implied by its parameter values, which are extremely close to those of the basic reduced form model, the *low* k^F case has 99.5 percent of its total capital stock being allocated to the production of intermediate goods. Consequently, the results for this case are indistinguishable from those in table A3 and we do not report them again but use those already reported here.

Results for the *high* k^F case, where only 11.2 percent of total capital is used in the intermediate goods sector, are reported in table A4. A comparison of tables A3 and A4 indicate that the share of capital allocated to the production of intermediate goods has almost no implication for the behavior of either the inventory or the control model. Importantly, the difference in the volatility of GDP 7.3 basis points in the *low* k^F case and 7.2 basis points in the *high* k^F where capital is much less important in the production of intermediate goods. The countercyclicality of the inventory-to-sales ratio is unaffected by the placement of capital.

These results motivate our conclusion that, in our (S,s) inventory model, the introduction of capital in the production of final goods is unlikely to substantially alter the behavior of the model. In the *low* k^F case, the slow accumulation of capital dampens changes in the production of intermediate goods and offsets most of the aggregate effects of inventory investment. In the *high* k^F case, capital is less important in constraining changes in the supply of intermediate goods. However, the level of capital in the production of final goods is now an important determinant of the marginal product of intermediate goods. As a result, the slow accumulation of capital now directly limit the demand for intermediate goods in current production which, in equilibrium, slows their production as before.

A.2 Alternate reduced form model

In closing this section, we discuss an alternate basic reduced-form model. The model presented above assumes final goods inventories as a factor of production. Thus, in contrast to our (S,s) inventory model, inventories of final, rather than intermediate, goods are held. We have also examined a model where inventories of intermediate goods are a factor of production.

While any reduced-form model presents problems of interpretation, these are substantially greater for this model. Here, inventories are depleted to supply intermediate goods in production. However, they are also a direct factor of production. Thus intermediate goods are productive when they are directly used in production, and also while they are held as inventories prior to use. The final goods version of the model presented above seems preferable in that it does not imply goods are productive both when they are used and before they are used. Nonetheless, for completeness, we mention some results for a version of this alternate model where, as in section A, capital is used only in the production of intermediate goods

When inventories of intermediate goods are a factor of production, and shocks are to intermediate goods (as in the baseline (S,s) inventory model in section 5 of the paper,) then this model exhibits a percent standard deviation of GDP that is 8 basis points higher than in its counterpart model without inventories. When we instead consider shocks to final goods, this model implies a 7 basis point *fall* in the variability of GDP relative to its counterpart without inventories. By contrast, in our (S,s) inventory model, the variability of GDP, under shocks to final goods production, was 5 basis points higher than in the control model without inventories. Thus the qualitative response of fluctuations in GDP to the presence of inventories in this reduced-form model is inconsistent with that seen in the (S,s) model.

Table A1: Sensitivity cases part 1*

	GDP	FS	NII	IS ratio	L	N	X	M	q	I
A: Final goods shock										
control rel. SD	**1.572**	1.000			0.642	0.642	0.413	0.413	0.593	7.175
inventory rel. SD	**1.626**	1.071	0.157	0.709	0.534	0.709	0.399	0.517	0.535	8.577
control corr		1.000			0.967	0.967	0.992	0.992	0.996	0.967
inventory corr		0.991	-0.388	**-0.975**	0.977	0.922	0.916	0.955	0.998	0.929
B: Economywide shock										
control rel. SD	**1.572**	1.000			0.642	0.642	0.808	0.808	0.208	7.175
inventory rel. SD	**1.619**	0.997	0.081	0.735	0.625	0.646	0.807	0.788	0.185	7.773
control corr		1.000			0.967	0.967	0.996	0.996	0.940	0.967
inventory corr		0.997	0.075	**-0.966**	0.980	0.943	0.988	0.995	0.955	0.947
C: Preference shock										
control rel. SD	**1.602**	1.000			1.560	1.560	0.980	0.980	0.023	0.990
inventory rel. SD	**1.692**	1.026	0.051	0.818	1.484	1.519	0.941	0.994	0.029	1.799
control corr		1.000			1.000	1.000	1.000	1.000	0.871	0.967
inventory corr		0.999	-0.494	**-0.999**	1.000	0.999	0.999	0.999	0.949	0.899

* In rows 1 - 2 of each panel, column 1 reports percent standard deviations of GDP; remaining columns are standard deviations relative to GDP. All series are log HP-filtered, except NII, which is detrended as a share of GDP, and IS ratio, which is detrended in levels. Rows 3 - 4 of each panel report contemporaneous correlations with GDP. Contemporaneous correlations between HP-filtered final sales and net inventory investment from inventory models are as follow: panel A: -0.510, panel B: -0.001, panel C: -0.531.

Table A2: Sensitivity cases part 2*

	GDP	FS	NII	IS ratio	L	N	X	M	q	I
A: High RTS Case 1 (higher labor share using θ_N)										
control rel. SD	**1.590**	1.000			0.654	0.654	1.502	1.502	0.513	7.332
inventory rel. SD	**1.595**	0.883	0.125	0.865	0.804	0.561	1.587	1.346	0.493	6.729
control corr		1.000			0.967	0.967	0.996	0.996	-0.965	0.965
inventory corr		0.999	0.948	**-0.939**	0.972	0.970	0.997	0.991	-0.965	0.967
B: High RTS Case 2 (higher capital share using θ_N)										
control rel. SD	**1.552**	1.000			0.701	0.701	1.459	1.459	0.470	5.595
inventory rel. SD	**1.535**	0.910	0.095	0.904	0.811	0.631	1.523	1.337	0.460	5.386
control corr		1.000			0.976	0.976	0.996	0.996	-0.965	0.978
inventory corr		0.999	0.949	**-0.964**	0.980	0.976	0.996	0.993	-0.966	0.978
C: High RTS Case 3 (higher capital share using θ_M)										
control rel. SD	**1.552**	1.000			0.701	0.701	1.358	1.358	0.366	5.595
inventory rel. SD	**1.548**	0.912	0.093	0.816	0.783	0.622	1.403	1.245	0.357	5.281
control corr		1.000			0.976	0.976	0.998	0.998	-0.971	0.978
inventory corr		1.000	0.958	**-0.960**	0.979	0.979	0.998	0.995	-0.972	0.980

* In rows 1 - 2 of each panel, column 1 reports percent standard deviations of GDP; remaining columns are standard deviations relative to GDP. Rows 3 - 4 of each panel report contemporaneous correlations with GDP. Contemporaneous correlations between HP-filtered final sales and net inventory investment from inventory models are as follow: panel A: 0.933, panel B: 0.938, panel C: 0.949.

Table A3: Reduced-Form Model of Inventories *

	GDP	FS	NII	IS ratio	consump.	total hours	invest.
A: standard deviations relative to GDP							
control	**1.717**	1.000			0.411	0.643	7.183
inventory	**1.790**	0.906	0.153	0.438	0.385	0.652	6.214
B: contemporaneous correlations with GDP							
control	1.000	1.000			0.921	0.968	0.971
inventory	1.000	0.992	0.664	**- 0.931**	0.912	0.972	0.973

* Column 1 of panel A reports percent standard deviation of GDP. All series are log HP-filtered, except NII, which is detrended as a share of GDP, and IS ratio, which is detrended in levels. Inventory model assumes stocks of final goods are required in final production. The share to this factor is set to match average inventory-sales ratio of 0.7155. Other parameters set to match calibration targets in main text. Contemporaneous correlation between final sales and net inventory investment: 0.565. Control model removes inventories by setting the share to final goods stocks at 0.0001.

Table A4: Reduced-Form Inventories with Capital in Both Sectors – *High k^f case* *

	GDP	FS	NII	IS ratio	consump.	total hours	invest.
A: standard deviations relative to GDP							
control	**1.717**	1.000			0.411	0.643	7.185
inventory	**1.789**	0.906	0.153	0.438	0.385	0.652	6.214
B: contemporaneous correlations with GDP							
control	1.000	1.000			0.921	0.968	0.971
inventory	1.000	0.992	0.664	**- 0.931**	0.912	0.972	0.973

* Column 1 of panel A reports percent standard deviation of GDP. All series are log HP-filtered, except NII, which is detrended as a share of GDP, and IS ratio, which is detrended in levels. Inventory model assumes stocks of final goods are required in final production. The share to this factor is set to match average inventory-sales ratio of 0.7155. Share to capital in final goods production set to 0.165. Remaining parameters set to match calibration targets in main text. Contemporaneous correlation between final sales and net inventory investment: 0.565. Control model removes inventories by setting the share to final goods stocks at 0.0001.