Additional Appendices for 'Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity'

Aubhik Khan and Julia K. Thomas

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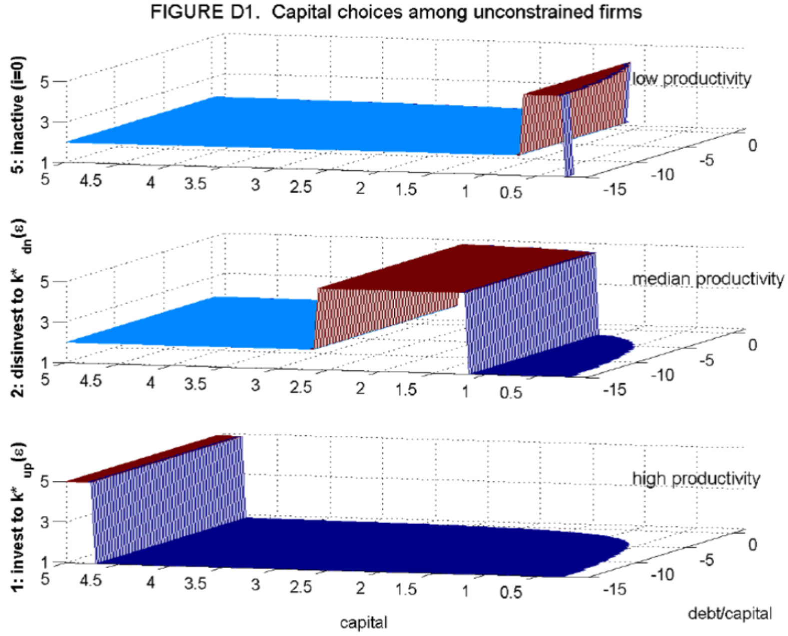
Appendix D. Steady state capital decisions

Figure D1 illustrates the pure effects of the irreversibility in cases where it does not interact with the financial friction. It summarizes the capital choices made by unconstrained firms entering the period with a given capital (measured on the x-axis) and debt (measured on the z-axis), conditional on current productivity. The top panel considers firms entering with the lowest productivity value, the middle panel is those with the median value, and the bottom panel is those with the highest productivity. The y-axis in each panel reports an indicator variable that takes the value 1 for unconstrained firm states implying positive investment to the upward target capital consistent with current productivity, the value of 2 for unconstrained states implying negative investment to the relevant downward target, and the value of 5 for those implying investment inactivity. (The right, rear areas left blank are combinations of $(k, b/k)$ where firms are not unconstrained.)

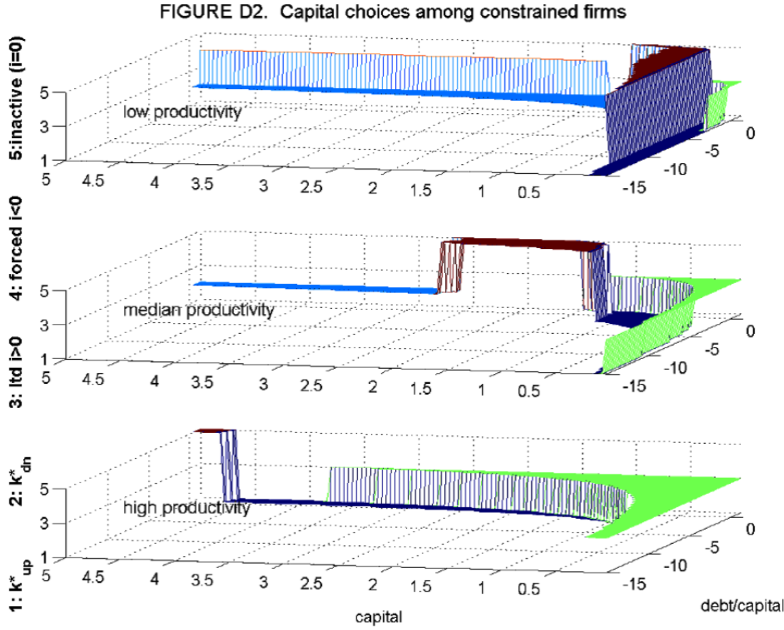
FIGURE D1. Capital choices among unconstrained firms



Looking from the top panel downward, the region of $(k, b/k)$ where firms adopt their upward target capital shifts into higher current capital levels, since rises in current productivity predict higher marginal product of capital schedules next period. To the left of these regions are areas with zero investment induced by irreversibility. While the loss associated with uninstalling capital in our economy is only 5 percent, it nonetheless makes some firms reluctant to shed capital. Those with higher productivities are more so, given persistence in ε and capital depreciation. As such, the inactivity region expands to higher capital levels as productivity rises, while the region associated with downward investment shrinks, then disappears from view by the bottom panel. On balance, of the 9 percent of firms in our stationary distribution that are (permanently) unconstrained, 35 percent adjust to the upward targets consistent with their productivities, 16 percent undertake negative investment to their downward targets, and 49 percent are inactive with respect to capital. Similar proportions hold for the 29 percent of firms that never face borrowing limits (no-constraint firms), since they optimally adopt the unconstrained firm capital rules; however, they are, on average, younger than unconstrained firms, so more of them invest.

Figure D2 is analogous to Figure D1. Conditional on current productivity, it illustrates the capital decisions taken by firms, this time considering those affected by both the real friction in our economy and the financial one. Such firms are 62 percent of the population in steady state, and are located in regions of the $(k, b/k)$ space to the right and back where capital is low or debt is high. (Blank areas in the foreground are combinations of $(k, b/k)$ where firms are unconstrained.)

FIGURE D2. Capital choices among constrained firms



Because constrained firms' capital choices are largely determined by life-cycle factors stemming from collateral constraints, they tend to avoid negative investment. The indicator 3 reflects firm states implying positive investment to the maximum affordable capital, $\overline{k}_u(k, b, \varepsilon)$. These correspond to firms with higher productivity, comparatively low capital and comparatively high debt; they make up 28 percent of constrained firms in our model's steady state and are the only firms facing a currently binding borrowing limit. Just left and in front of that region, firms with slightly higher capital (or slightly lower debt) adjust to their upward capital targets, $k_u^*(k, b, \varepsilon)$. This region, reflected by the indicator 1, expands into higher values of capital as ε rises, since the target itself rises. In the stationary distribution, roughly 34 percent of constrained firms are of this type. Further left in each panel are firms selecting investment inaction due to the irreversibility (indicator 5), and then those whose capital is sufficiently high relative to their productivity that they disinvest to their downward targets (indicator 2). These categories represent 30 and 9 percent of constrained firms in our model's stationary distribution. Finally, the sliver of high debt states with indicator value 4 in the top panel are states where low productivity firms must undertake downward capital adjustment to repay their debts; this is a rare situation affecting fewer than 1 percent of firms in the stationary distribution.

Appendix E. Special case models with altered frictions

No one has ever examined a DSGE heterogeneous firm model with both real and financial frictions before, so we demonstrate in this appendix how business cycles in our economy are affected by each individually. We begin by presenting moments drawn from $10,000$ period simulations of special case versions of our model with one or both frictions removed. Table E1 eliminates real frictions (setting $\theta_k = 1$), Table E2 eliminates financial frictions (setting $\zeta = 500$ in all periods and verifying all firms are always unconstrained), while Table E3 eliminates both real and financial frictions. Throughout these exercises, all other parameters are held fixed at the baseline values from Table 1. Naturally, by comparison to our full model simulation in Table 2 of the main text, these tables confirm that each friction reduces average output, consumption and capital. Moving to consider second moments, again relative to the full model results in the main text, there are some differences here worthy of brief comment.

TABLE E1. Business cycles without real frictions

$x =$	Y	C	I	N	K	r
mean(x)	0.585	0.495	0.090	0.330	1.385	0.042
σ_x/σ_Y	(2.091)	0.490	4.419	0.616	0.507	0.441
$corr(x, Y)$	1.000	0.877	0.949	0.926	0.083	0.683

Comparing Tables 2 and E1, we see output volatility rise when the real friction is removed. Despite this, the representative household is more effective in smoothing consumption. Absent capital irreversibility, aggregate investment is more responsive to shocks and hence so is labor supply. This evidence that firm-level capital irreversibilities dampen changes in aggregate investment is consistent with findings by Bertola and Caballero (1994).

Comparing Table E2 with the full model results in Table 2, GDP volatility falls when financial frictions are eliminated simply because this change necessarily rules out any effects of credit shocks. Comparing Table E2 instead with Table 3 (our otherwise full model without credit shocks), we see that GDP responds just marginally more to TFP shocks if credit frictions are removed. More generally, the typical business cycle in our economy is relatively impervious to an ordinary, ongoing degree of financial frictions so long as aggregate fluctuations are driven by shocks to productivity alone. This observation is consistent with previous findings in the literature that financial frictions have minimal consequence for real responses to nonfinancial shocks in quantitative DSGE models; see Cordoba and Ripoll (2004) and Kocherlakota (2000).

TABLE E2. Business cycles without financial frictions

$x =$	Y	C	I	N	K	r
mean(x)	0.598	0.497	0.101	0.336	1.428	0.042
σ_x/σ_Y	(2.011)	0.496	3.795	0.570	0.477	0.444
$corr(x, Y)$	1.000	0.927	0.968	0.946	0.071	0.676

Comparing Tables E1 and E3, starting from a setting without real frictions, output volatility falls with the removal of the financial friction. This is primarily because, again, credit shocks are meaningless absent financial frictions. When we eliminate the volatility and non-monotone production responses associated with credit shocks, consumption becomes less volatile and more correlated with contemporaneous production. Investment is also less volatile and more correlated with output, and thus so is hours worked. Each observation is easily understood in light of the impulse responses from section VI-C of the text.

TABLE E3. Business cycles with no frictions

$x =$	Y	C	I	N	K	r
mean(x)	0.610	0.511	0.099	0.333	1.530	0.042
σ_x/σ_Y	(2.034)	0.479	4.046	0.591	0.462	0.434
$corr(x, Y)$	1.000	0.917	0.967	0.950	0.057	0.679

Finally, Table E4 provides a peak-to-trough summary of our model's response to a 4-period credit crisis under alternative assumptions regarding capital reversibility. Note that irreversibility alters our economy's response to financial shocks, particularly where investment and employment are concerned. Absent the calibrated real friction (comparing baseline to perfect reversibility), productive inputs are more adversely affected by a credit shock and thus so is GDP. When we compare our calibrated baseline model ($\theta_k = 0.954$) to a setting where irreversibility is so severe ($\theta_k = 0.75$) that firms never actively disinvest ($i/k \leq -0.01$), the differences are dramatic.

TABLE E4. Peak-to-trough declines: Varying capital reversibility

	Trough	GDP	I	N	C	TFP	Debt
perfect reversibility	4	4.88	25.48	3.80	1.12	1.55	14.47
baseline	4	4.38	21.88	3.42	0.99	1.30	25.96
severe irreversibility	4	2.95	12.21	2.00	0.98	0.67	19.26

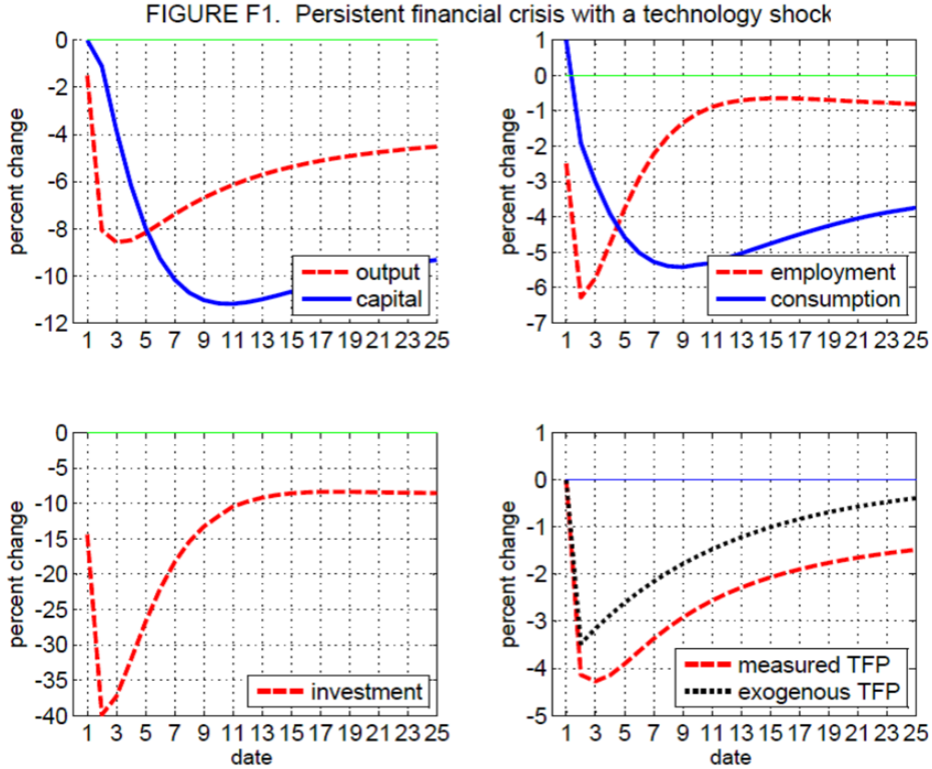
NOTE.– NOTE.– Row 2 is 4-period credit crisis (with full ζ recovery in period 5) in the baseline calibrated model with $\theta_k = 0.954$. Row 1 is the counterpart in a version of the model with $\theta_k = 1.0$ (all other parameters fixed). Row 3 is the counterpart in a model with $\theta_k = 0.75$ (all other parameters fixed). All declines measured at GDP trough date in column 2.

The trough date in column 1 is unaffected by variation in θ_k, while the decline in debt responds nonmonotonically. Elsewhere, the credit crisis has a more negative impact on GDP, investment, employment and TFP the more reversible is capital. Conversely, households are more successful in shielding their consumption the closer the investment technology is to frictionless. In the case of $\theta_k = 1$, consumption continues to fall after GDP troughs in each model, as in Figure 9; ultimate consumption declines for rows 1, 2, and 3 are 2.57, 2.38 and 1.83 percent, respectively.

Appendix F. Credit crisis compounded by real shock

Here, we briefly consider what implications the prolonged financial crisis from Figure 6 can have if its onset is followed by a 1 standard deviation (3.47 percent) negative technology shock. As seen in the lower right panel of Figure F1, we assume that the exogenous component of TFP falls one year after the financial shock hits, and thereafter gradually reverts to its mean. Were credit markets functioning as normal when the TFP shock appeared, output would fall 5.1 percent, labor would fall 2.7 percent, and the half-life of the output response would be roughly 7 years. While the overall declines in quantities here are larger and more protracted, they are no more so than would be expected once we account for the effects of the credit shock in Figure 6. Thus, we conclude that the TFP shock has little interaction with the credit shock.

FIGURE F1. Persistent financial crisis with a technology shock



NOTE.– Credit shock starting in date 1 with a persistent 1 standard deviation drop in z starting in date 2. Expectations consistent with calibrated processes. Productivity reverts at expected rate; credit variable remains at crisis level through period 25. Y-axes measure percentage deviations from simulation means.

References

Bertola, Giuseppe and Ricardo J. Caballero. 1994. “Irreversibility and Aggregate Investment.” *Review of Economic Studies* 61: 223-46.

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