Productive Misallocation and International Transmission of Credit Shocks

Yuko Imura

Bank of Canada

Julia K. Thomas*

Ohio State University

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ABSTRACT

We develop an asymmetric two-country equilibrium business cycle model to study the role of international trade in transmitting the real effects of financial shocks across borders. Our heterogeneous firms have differing needs for external finance and face occasionally binding collateral constraints hindering their investments, while input-output linkages facilitate trade in both final goods and intermediate inputs. When confronted with global financial shocks, our model predicts that the recession in a large economy calibrated to the U.S. appreciably alters the recession in its smaller trade partner calibrated to Canada, with distinct investment dynamics driving the transmission. The reverse does not hold.

Keywords: Credit crisis, international trade collapse, capital misallocation

*Imura: yimura@bankofcanada.ca, Thomas: mail@juliathomas.net. The views expressed in this paper are our own and do not necessarily represent those of the Bank of Canada.

1 Introduction

United States financial market disruptions that began as domestic distress rapidly spread during the 2007-09 financial crisis, triggering sharp contractions in real activity across advanced and emerging economies. As financial and economic conditions deteriorated, countries around the world exhibited a high degree of cross-country synchronization in aggregate quantity variables.[1] A number of empirical studies emerged from the crisis, uncovering evidence on the nature of shocks originating in financial markets and their severe consequences. Meantime, theoretical analyses of these shocks explored various financial market frictions to isolate powerful propagation mechanisms through which financial shocks might generate large downturns in aggregate economies.

An abrupt collapse in international trade accompanied the cross-country comovement of real and financial variables over the financial crisis.[2] World merchandise trade fell by 38 percent in nominal terms between 2008Q2 and 2009Q1 according to World Trade Organization international trade statistics, and some empirical studies link this to fallout from the global financial crisis. Bems, Johnson and Yi (2010), Eaton, Kortum, Neiman and Romalis (2016) and Bussière et al. (2013), among others, have ascribed the trade collapse to large reductions in expenditures on traded goods. For example, Behrens, Corcos and Mion (2010) attribute more than half of the fall in Belgian firms' exports to contractions in aggregate demand (GDP growth rates) in their destination economies.

When demand contracts sharply in a country's close trade partners, the resulting decline in goods trade can have significant implications for its aggregate economy. Bems, Johnson and Yi (2010) use an international input-output framework and estimate that 27 percent of the fall in U.S. aggregate demand during the crisis was borne by foreign producers; they find that Canada and Mexico bore a particularly large share of the contraction, in what amounted to a 2.3 percent reduction in their economies' GDP. Beyond such demand-based transmission, some authors offer empirical evidence of supply-side effects of the financial crisis on exporters.[3] While financial linkages certainly had a critical role in propagating financial distress across countries, the large decline in international trade during this period suggests trade linkages also may have had an important role in transmitting the negative consequences of the U.S. financial crisis throughout the world.

This paper uses an equilibrium international business cycle model to examine trade linkages as a contributing factor in spreading the real consequences of financial disruptions from one country to

[1] See, for example, Kollmann, Enders and Müller (2011), Ueda (2012), and Perri and Quadrini (2018).

[2] Bems, Johnson and Yi (2013) offer an excellent survey of works examining the collapse in trade during the crisis.

[3] Chor and Manova (2012) link export performance during the crisis period to exporters' financial conditions at the country level, while Behrens, Corcos and Mion (2012) and Bricongne et al. (2012) establish these links at the firm level for Belgian and French exporters, respectively.

its trade partners. We extend the model of Khan and Thomas (2013), wherein firms heterogeneous in capital and financial position face persistent idiosyncratic productivity shocks and occasionally binding borrowing limits, to a multi-sector, open economy setting with two asymmetric countries. As in Khan and Thomas (2013), our firms' investments in physical capital may be financed using internal funds and one-period debt, but debt choices are limited by a collateral constraint anchored on their existing capital stocks. Exogenous firm death and birth ensure there are always some firms in each country that cannot borrow what they would require to invest to the efficient capital stocks consistent with their productivities. The resulting capital misallocation lowers aggregate productivity, employment, investment and GDP even in ordinary times, and it is through this channel that a credit shock tightening collateral constraints can cause a domestic recession.

While firms in our manufacturing sector are largely patterned on those in Khan and Thomas (2013), we introduce an interdependency between their production and tradeable goods sectors in line with salient regularities reported in the recent trade literature. There, cross-country production linkages have been emphasized as an increasingly prominent aspect of international business cycles, with trade in intermediate inputs now accounting for a large share of international trade.[4] With that in mind, our model features a roundabout structure so that, in effect, part of output is used as production inputs by all firms in the economy, at home and abroad. Manufacturing firms use materials, alongside capital and labor, to produce an output they sell to a domestic wholesale sector; there, manufactured goods are combined with labor to produce tradeable goods, destined for final goods production and tradeable goods used to produce materials. Interpreting the latter as intermediate inputs, our model captures the extent of intermediate inputs use relative to firms' value added, as well as the share of imported content in these inputs.

We calibrate our two-country economy to the United States and Canada, replicating their trade linkages and the shares they have in each country's GDP, as well as their relative production sizes. The strong trade relationship between these two countries, alongside their geographic proximity, makes this pairing a natural candidate for our analysis focused on the role of bilateral trade linkages in transmitting the effects of financial shocks. The World Bank's World Integrated Trade Solution data show Canada was the top destination for U.S. exports in 2007, accounting for 21.4 percent of total U.S. exports, and it was the second largest source of imports into the U.S. (15.7 percent), closely following China (16.9 percent). At the same time, 79 percent of Canadian exports went to the U.S., and the U.S. supplied 54.2 percent of Canada's imports.[5] Our calibrated model reflects the

[4] See, for example, Burstein, Kurz and Tesar (2008) and di Giovanni and Levchenko (2010).

[5] Mexico was the second largest U.S. export destination in 2007, with 11.7 percent of U.S. exports. The second largest export destination for Canada was the U.K., with 2.8 percent of Canadian exports. Behind the U.S. 54.2

significant difference in each country's reliance on its trade partner, with the U.S. having minimal exposure to the Canadian economy relative to the size of its domestic activity and Canada having substantially greater exposure to the U.S. economy relative to its domestic activity. As such, it is a useful laboratory to study the transmission of a U.S. financial shock to its smaller individual trading partners and vice versa, with greater realism than possible using a symmetric two country calibration commonly adopted in the open economy business cycle literature. Another advantage in calibrating to these two advanced, neighboring economies is that it avoids some economic considerations absent from our model, such as exchange rate policy, trade policy, transportation costs and lags, that themselves may have non-trivial quantitative effects on international trade transmission.

We use our model first to consider the domestic impact of simultaneous credit shocks in the two trading economies, selecting the size and persistence of shocks so that our predicted paths of debt match those of real nonfinancial business debt in the U.S. and Canada over the financial crisis. This unanticipated tightening of collateral constraints distorts the allocation of capital across firms in each country, inducing large, persistent declines in endogenous aggregate productivity and hence investment, employment and production.[6] More concretely, with shocks calibrated to conservative measures of the declines in overall business lending and absent other significant events unfolding over this episode (e.g., housing crisis and failures in financial firms), our model explains roughly 40 and 20 percent of the observed peak-to-trough GDP declines in the U.S. and Canada, respectively.

Delving beneath the aggregate series, we demonstrate the firm-level consequences of these shocks with particular attention to the impact on firm life cycles. We show that young firms are disproportionately affected by financial shocks, especially those with high relative productivities. Born comparatively small, young firms tend to be cash-poor and so more reliant on external funds. Tight credit conditions drive persistently raised inefficiency in their investments, slowing their life cycle growth relative to that in ordinary times. This disparate incidence by age increases misallocation, as all firms draw from the same productivity distribution. Moreover, among the most affected firms, we show credit shocks particularly distort the investments of those with higher productivity, since their efficient capital stocks are larger. This further tilts the distribution of capital, and so production, toward less productive firms. These problems do not vanish quickly with the restoration of credit conditions, as all cohorts born during and shortly before the crisis see their life cycle growth protracted; thus, the aggregate recovery is gradual.

We next examine the transmission of each country's financial shock recession to its trade partner,

share of Canada's imports, China and Mexico supplied 9.4 and 4.2 percent, respectively.

[6] See Khan and Thomas (2013) for a discussion of the mechanics of such shocks in a closed economy setting.

quantifying the relative impact of domestic versus foreign shocks in driving aggregate dynamics over the downturn and thereafter. We show that the credit shock in the large economy calibrated to the United States appreciably delays the recession trough and subsequent recovery in its smaller trade partner calibrated to Canada. Comparing to a situation where only the small economy directly experiences a shock, the financial shock in its large trading partner initially puts upward pressure on investment in the small country. This positive spillover partly offsets the recessionary effects of the domestic shock, softening the decline in GDP over early dates of the recession and delaying its trough by six quarters. Over time, however, the prolonged contraction in the large economy persistently erodes demand for the small economy's exports, inducing a sizeable reversal in its investment. This drives anemic recovery in the small economy, extending the half-life of its GDP response by a decade in comparison with the recovery path when it alone experiences a shock.

In contrast, our model predicts that a financial shock originating in the small economy has quite minimal consequences for investment, employment and production in the large economy. Comparing to a situation where only the large country directly experiences a shock, we do find that its exports decline further when the small country also suffers a financial shock. However, trade constitutes only a minor fraction of the large country's GDP in our calibration to the U.S. and Canada, so the small economy's reduced import demand has a negligible effect on its aggregate production. This prediction of our model is consistent with findings in the empirical literature that business cycle synchronization correlates with the strength of trade linkages across countries.

Finally, we compare international transmission of financial shocks to that of productivity shocks. Focusing on the consequences of a large-economy shock for the small economy absent any domestic shock, we decompose the GDP response into its expenditure-side components. We find that the transmission of the financial shock to the small economy's aggregate production is mainly driven by adjustments in investment, while transmission of the productivity shock occurs mainly through adjustments in consumption. This pattern reflects the differing transmission channels inside the large economy in response to domestic financial versus productivity shocks.

Because a financial shock drives a domestic recession mainly through its effects on firms' investments, consumption's share in the GDP decline following that shock is modest relative to its share in the downturn brought on by a productivity shock. In our complete markets setting, the resulting real exchange rate adjustment is a direct function of the change in the originating country's relative per-capita consumption. Thus, the decline in the small country's consumption in proportion to its overall GDP change is limited by its trading partner's modest consumption decline. Though we do not explicitly model direct exposure of domestic agents to foreign financial frictions, and hence financial market contagion, this indicates that trade linkages can imply a more prominent role for investment adjustment in the trade partner's economy in transmitting local financial shocks abroad.

The remainder of the paper is organized as follows. Section 2 reviews the literature related to our study. We describe the model economy in Section 3 and discuss its calibration in section 4. Section 5 presents our quantitative results, and section 6 concludes.

2 Related literature

Our analysis of the real economic consequences of financial shocks builds on advancements in the closed-economy quantitative financial frictions literature prompted by the Great Recession of 2007-09. In particular, we draw from the work of Khan and Thomas (2013) wherein collateralized borrowing constraints and firm heterogeneity yield a powerful propagating channel through which an aggregate tightening of collateral constraints heightens capital misallocation, thereby lowering aggregate productivity and in turn discouraging savings and hours worked. We extend their model to an international real business cycle framework and introduce input-output linkages among firms to capture international trade in both final goods and intermediate inputs.

Investment contractions play a central role in our extension of Khan and Thomas' model, while other studies emphasize the labor demand channel in propagating shocks to financial frictions. Arellano, Bai and Kehoe (2019) examine an uncertainty shock model where firms take on defaultable debt to finance their employment bills before observing idiosyncratic productivity shocks. Because agency frictions reduce firms' incentives to self-insure, an uncertainty shock raising the volatility of their productivity draws effectively operates as a financial shock by amplifying default risk. The resulting rises in loan interest rates and firm liquidations discourage hiring, and can cause large reductions in aggregate employment and production similar to those seen over the Great Recession. Jermann and Quadrini (2012) study a representative firm model where working capital costs are financed using debt and equity, and the firm faces an enforcement constraint to ensure it repays its loans. Given a tax preference for debt, this constraint always binds, and convex costs of adjusting equity prevent the firm from fully compensating with equity when a negative shock tightens it, driving up the effective cost of labor. Through this channel, the authors show that financial shocks can generate the observed dynamics of output and employment as well as debt and equity flows during the financial crisis, accounting for almost half of the output volatility.

Whereas labor market inefficiencies are central in Arellano, Bai and Kehoe (2019) and Jermann and Quadrini (2012), the source of our model's misallocation is the interaction of a backward-looking collateral constraint with persistent firm-level heterogeneity in productivity and size that together drive investment inefficiencies. Examining a similar setting with constant returns production and i.i.d. shocks where firms' debts may not exceed a given fraction of their capital choices, Buera and Moll (2015) show that financial shocks can cause differing aggregate distortions depending on the source of heterogeneity. When a financial shock tightens the collateral constraint in their model, the resulting productive distortion may be an efficiency (TFP), investment or labor wedge depending on whether firms differ in their productivities, investment costs or labor recruitment costs.

As with many other such studies, our adoption of collateralized borrowing constraints as the source of financial frictions follows from the theoretical work of Kiyotaki and Moore (1997), who showed collateral constraints can amplify and propagate shocks to the value of collateral, generating large fluctuations in asset prices and production. An alternative source of frictions widely used in the literature is the financial accelerator mechanism introduced by Bernanke, Gertler and Gilchrist (1999). Their work builds on Townsend's (1979) costly state verification model where an information asymmetry between lenders and borrowers yields an external finance premium inversely related to borrowers' net worth. More recently, Kiyotaki and Moore (2019) introduce another type of friction, a resaleability constraint limiting the equity entrepreneurs can issue to raise funds for investment. The resulting illiquidity of assets gives rise to a role for money as a source of liquidity, offering new insights for monetary policy responses to liquidity shocks. Del Negro, Eggertsson, Ferrero and Kiyotaki (2017) find that liquidity shocks to the resaleability constraint can account for a large share of the GDP contraction and the entire fall in inflation during the Great Recession.[7]

Alongside these developments in the closed-economy analysis of financial frictions, the strong synchronization of business cycles across countries during the financial crisis energized a literature exploring international financial contagion and business cycle comovement. Devereux and Sutherland (2011), Dedola and Lombardo (2012), Perri and Quadrini (2018), Yao (2019) and Devereux and Yu (2020) examine international transmission of financial shocks through financial contagion arising from domestic agents' direct exposure to foreign assets.[8] In their models, domestic investors can access foreign assets subject to financial frictions, and a financial shock in one country worsens borrowing conditions for not only domestic investors but also foreign investors exposed to that country's assets, thereby inducing international comovement in investment and production. Our

[7] Bigio (2015) and Kurlat (2013) endogenize liquidity constraints by introducing asymmetric information about the quality of assets. Ajello (2016) introduces financial intermediaries in the model of Kiyotaki and Moore (2019), and shows that shocks to intermediation costs account for a large fraction of output and investment volatility.

[8] Earlier studies introduce financial frictions in two-country models to study real and nominal shock transmission. See, for example, Gilchrist (2004), Iacoviello and Minetti (2006), Faia (2007), and Devereux and Yetman (2010).

paper is complementary to this body of work in analyzing how an additional transmission through international goods trade can affect the economies' responses to financial shocks.[9]

More broadly, our work contributes to a large literature studying the cross-country transmission of shocks in international business cycle models, starting with Backus, Kehoe and Kydland (1992, 1994) and Baxter and Crucini (1995). Kose and Yi (2006) explore whether a standard international business cycle model can account for the observed relation between bilateral trade intensity and business cycle comovement. They find the model succeeds in generating a positive relationship between trade linkages and output comovement, though the implied elasticity is substantially lower than in the data. We do not quantify this relationship in our analysis, but our results are consistent with theirs in demonstrating that an economy more reliant on international trade in proportion to its GDP is more susceptible to financial shocks originating in its trade partner's economy.

The inclusion of input-output linkages within and across countries in our model follows recent work emphasizing the growing empirical importance of trade in intermediate inputs, particularly in light of their role during the trade collapse in the midst of the global financial crisis. Bems, Johnson and Yi (2011) report that trade in intermediate inputs accounted for over 40 percent of the fall in total trade during this time. Burstein, Kurz and Tesar (2008) embed a vertically integrated production structure in a two-country business cycle model, finding that cross-country production linkages with complementarity in domestic and foreign inputs strengthen the relationship between trade and business cycle comovement. Johnson (2014) introduces cross-country, cross-sectoral production linkages in a multi-country real business cycle model capturing the complex web of production interdependency across sectors and countries. He shows that the model can predict the strong correlation between trade linkages and output comovement only when shocks are correlated across countries, and examines alternative elasticities affecting this relationship.

3 Model

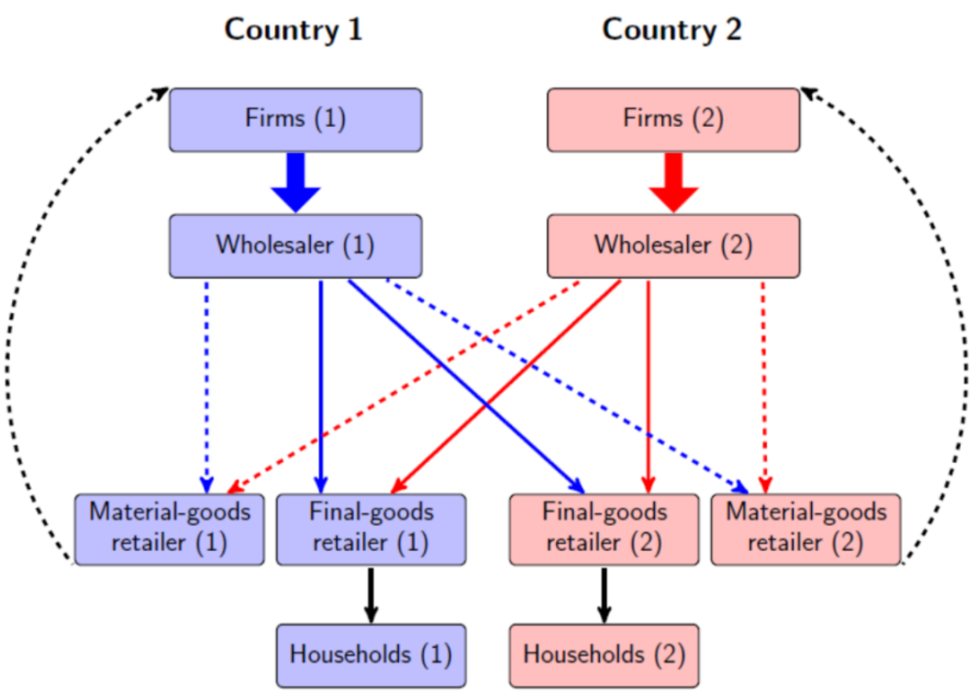
We consider two countries, $c = 1, 2$, which are distinguished in size by their time-invariant measures of households, Ψ_c. The countries are symmetric in most other respects, but their differing sizes will imply large differences in international trade exposure in our calibration. All markets are perfectly competitive, prices are flexible, and we assume complete international financial markets.

Each country houses a set of identical infinitely-lived households, each with access to state-

[9] A number of recent papers have also focused on an explicit role for global banks in transmitting financial shocks across borders. See, for example, Mendoza and Quadrini (2010), Ueda (2010), Kollmann, Enders and Müller (2011), Kalemli-Ozcan, Papaioannou and Perri (2013), Kamber and Thoenissen (2013), and Kollmann (2013).

contingent nominal bonds. The production structure is a hybrid of supply chain and roundabout production. A representative final goods retailer combines domestically produced and imported wholesale goods to produce the good used for domestic consumption and capital investment. A representative wholesaler combines domestic labor and manufactured goods to supply wholesale goods to foreign and domestic final goods retailers, and to foreign and domestic material goods retailers that, in turn, supply materials to firms. Finally, each country has a time-invariant measure of heterogeneous manufacturing firms matching its household population size, and each such firm uses capital, labor and materials to supply manufactured goods to the domestic wholesaler.

MARKET INTERACTIONS



Manufacturing firms (hereafter, *firms*) produce a common domestic good, and they face both persistent country-specific aggregate total factor productivity shocks and idiosyncratic productivity shocks. Firms hire labor from domestic households, purchase intermediate inputs from domestic materials retailers and maintain their own capital stocks. Each invests goods bought from the domestic final-good retailer to augment its capital for the next period, and each can use one-period loans to help finance these purchases. A collateralized borrowing constraint in each country limits the debt any firm can take on as a function of its existing capital stock. Firms cannot circumvent the constraint by paying negative dividends. We also assume exit and entry at an exogenous rate each period to prevent all firms effectively outgrowing financial frictions in the long run.

The aggregate state of the world economy is A, where $A \equiv (S, Z)$ and $Z \equiv [z_1, z_2, \zeta_1, \zeta_2]$ is the exogenous state vector. The latter includes aggregate productivity, z_c, and the credit state ζ_c, in each country, $c = 1, 2$. Each country's ζ_c anchors the collateral constraints limiting firms' debt in proportion to their current capital. All exogenous state variables follow Markov chains; z takes on one of N_z values, ζ takes on one of N_ζ values, and the joint transition matrix Π^Z has elements $\{\pi^Z_{lm}\}$ representing $pr\{Z'_m|Z_l\}$ for $l, m = 1, \ldots, N_Z$, where $N_Z = N_z N_\zeta$.

Our model generates a time-varying distribution of firms over capital, ($k \in \mathbf{K} \subset \mathbf{R}_+$), debt ($b \in \mathbf{B} \subset \mathbf{R}$) and firm-specific productivity ($\varepsilon \in \mathbf{E}$) in each country. We summarize the distribution of firms at the start of a period in country c using the probability measure μ_c defined on the Borel algebra $\mathcal{S}$ generated by the open subsets of the product space, $\mathbf{S} = \mathbf{K} \times \mathbf{B} \times \mathbf{E}$ for each $c = 1, 2$. These distributions appear in the model's endogenous aggregate state vector, $S \equiv [\mu_1, \mu_2, B_1, B_2]$, alongside the state-contingent bonds held by households at the start of the period, B_c, in each country. All agents in the economy take as given the laws of motion determining Z' given Z, as well as the evolution of the endogenous state according to an equilibrium mapping $S' = \Gamma(A)$. We describe the preferences, technologies and optimization problems for each country c below, specifying country 1 versus country 2 where necessary for clarity or in defining notation.

3.1 Households

Each of Ψ_c identical households in a country is endowed one unit of time per period, values consumption and leisure according to a period utility function $u(C, 1 - N)$ and discounts future utility by the subjective discount factor $\beta \in (0, 1)$. Household wealth is composed of one-period shares in domestic firms, identified by the measures ξ_c, noncontingent real bonds corresponding to the debts of domestic firms, denoted by ϕ_c, and aggregate state-contingent nominal bonds. The aggregate bond is denominated in units of country 1 currency, and $B_c(A)$ denotes the country c per-capita nominal bonds redeemed in the current period given aggregate state A.

Each household in country 1 chooses its consumption, C_1, the total labor hours it supplies to the wholesaler and manufacturing firms, N_1, its shares in firms of each type with which to begin the next period, $\xi'_1(k', b', \varepsilon')$, and its real bonds for next period, ϕ'_1. The household also decides its purchases of state-contingent nominal bonds for next period, $B_1(A')$, which each guarantee one unit of country 1 currency if state A' is realized. Let $\varrho(A'; A)$ be the real price of one such bond in units of country 1 consumption goods, and define $\widetilde{\rho}_1(k, b, \varepsilon; A)$ as the dividend-inclusive values of current firm shares and $\rho_1(k', b', \varepsilon'; A)$ as the ex-dividend prices of new shares in a given firm type.

The country 1 real wage and aggregate price level are $w_1(A)$ and $P_1(A)$, respectively, and each real bond therein costs domestic households $q_1(A)$ units of consumption. Finally, let $G(A'|A)$

represent the conditional probability of realizing a given state A' next period, which is determined by $S' = \Gamma(A)$ and the exogenous transition probabilities for the elements of Z. A country 1 household's expected lifetime utility maximization problem then can be written as follows.

$$V_1^h(\xi_1, \phi_1, B_1(A); A) = \max_{C_1, N_1, \xi_1', \phi_1', B_1(A')} u(C_1, 1 - N_1) + \beta \int V_1^h(\xi_1', \phi_1', B_1(A'); A') G(dA'|A) \quad (1)$$

subject to:

$$\int \widetilde{\rho}_1 (k, b, \varepsilon; A) \, \xi_1 (d [k \times b \times \varepsilon]) + \phi_1 + \frac{B_1(A)}{P_1(A)} + w_1(A) N_1 \geq$$

$$C_1 + \int \rho_1 \left(k', b', \varepsilon'; A\right) \xi_1' \left(d \left[k' \times b' \times \varepsilon'\right]\right) + q_1(A) \phi_1' + \int \varrho(A'; A) B_1(A') dA'$$

Let $\lambda_1(A) = D_1 u(C_1, 1 - N_1)$ be the Lagrange multiplier on the budget constraint. Household efficiency conditions with respect to hours worked, shares and real bonds imply restrictions on the country 1 real wage, firm share prices and inverse loan price listed in (2) - (4). Efficiency conditions with respect to state-contingent nominal bonds yield the additional restrictions in equation 5.

$$w_1(A) = D_2 u(C_1, 1 - N_1) / \lambda_1(A) \quad (2)$$

$$\rho_1 \left(k', b', \varepsilon'; A\right) = \int \frac{\beta \lambda_1(A')}{\lambda_1(A)} \widetilde{\rho}_1 \left(k', b', \varepsilon'; A'\right) G(dA'|A) \quad (3)$$

$$q_1(A) = \int \frac{\beta \lambda_1(A')}{\lambda_1(A)} G(dA'|A) \quad (4)$$

$$\varrho(A'; A) = \frac{\beta \lambda_1(A')}{\lambda_1(A)} \frac{1}{P_1(A')} G(A'|A) \quad (5)$$

Households in country 2 solve a similar problem adjusted to reflect that their nominal bonds are not denominated in their currency. Let $Q(A)$ represent the current real exchange rate, the price of country 2 final output in units of country 1 final output. Each nominal bond held at the start of the period returns $\frac{1}{P_1(A)}$ units of country 1 output, each worth $Q(A)^{-1}$ units of country 2 consumption goods. Similarly, a nominal bond for next period state A' costs a country 2 household $\varrho(A'; A)$ units of country 1 output, each worth $Q(A)^{-1}$ units of country 2 consumption.

$$V_2^h(\xi_2, \phi_2, B_2(A); A) = \max_{C_2, N_2, \xi_2', \phi_2', B_2(A')} u(C_2, 1 - N_2) + \beta \int V_2^h(\xi_2', \phi_2', B_2(A'); A') G(dA'|A) \quad (6)$$

subject to:

$$\int \widetilde{\rho}_2 (k, b, \varepsilon; A) \, \xi_2 (d [k \times b \times \varepsilon]) + \phi_2 + \frac{B_2(A)}{P_1(A) Q(A)} + w_2(A) N_2 \geq$$

$$C_2 + \int \rho_2 \left(k', b', \varepsilon'; A\right) \xi_2' \left(d \left[k' \times b' \times \varepsilon'\right]\right) + q_2(A) \phi_2' + \int \frac{\varrho(A'; A)}{Q(A)} B_2(A') dA'$$

The country 2 household efficiency conditions imply restrictions on $w_2(A)$, $\rho_2 \left(k', b', \varepsilon'; A\right)$ and $q_2(A)$ mirroring those in equations 2 - 4, with $\lambda_2(A) = D_1 u(C_2, 1 - N_2)$, and restrict nominal bond prices to satisfy the equations in (7).

$$\varrho(A'|A) = \frac{\beta\lambda_2(A')}{\lambda_2(A)} \frac{Q(A)}{P_1(A')Q(A')} G(A'|A) \tag{7}$$

Combining (5) and (7), we have equations governing real exchange rate evolution across each date and state: $Q(A') = \frac{\lambda_2(A')}{\lambda_1(A')} \frac{\lambda_1(A)Q(A)}{\lambda_2(A)}$. Assuming an initial date zero wherein $\frac{\lambda_1(A^0)Q(A^0)}{\lambda_2(A^0)} = 1$, each period's real exchange rate is a ratio of the marginal utility of consumption in country 2 versus 1.

$$Q(A) = \lambda_2(A)/\lambda_1(A) \tag{8}$$

3.2 Retailers and international trade

Each country has two representative retailers. The first uses domestic and imported wholesale goods to make final goods for consumption and investment. The second uses domestic and imported wholesale goods to make materials used by domestic manufacturers. This roundabout technology assumption implies direct concurrent effects of international trade both on final goods output and upstream in the supply chain. We summarize the problems of retailers in each country $c = 1, 2$ at once below, using the notation $\widetilde{c}$ to represent the country's trade partner, $\widetilde{c} \neq c$.

3.2.1 Final goods retailers

The final goods retailer in country c combines per-capita domestic wholesale goods, h^{Dc}, with those exported from country $\widetilde{c}$, $h^{X\widetilde{c}}$, in a CES technology to make per-capita final goods, H_c:

$$H_c = \left[\theta_{Hc} \left(h^{Dc}\right)^{\frac{\rho-1}{\rho}} + (1-\theta_{Hc}) \left(h^{X\widetilde{c}}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}, \tag{9}$$

where ρ is the (Armington) elasticity of substitution between domestic and foreign inputs, and θ_{Hc} is a country-specific relative weight on domestic inputs (home bias). It sells its output at price $P_c(A)$ to domestic households for consumption and to manufacturing firms for investment.

All wholesale goods prices are dominated in the currency of the country where they are sold. Let $P_h^{D1}(A)$ and $P_h^{X2}(A)$ be the nominal prices the country 1 final goods retailer pays for domestic and foreign inputs, respectively, and let $P_h^{D2}(A)$ and $P_h^{X1}(A)$ be the nominal prices the country 2 final goods retailer pays for domestic inputs and those exported from country 1. Put more succinctly, the country c retailer pays $P_h^{Dc}(A)$ and $P_h^{X\widetilde{c}}(A)$ for its domestic and foreign inputs.

The final goods retailer solves the static problem in (10) given the technology in (9), with resulting per-capita conditional factor demands in (11).[10]

$$\max_{h^{Dc}, h^{X\widetilde{c}}} \; \Psi_c[P_c(A)H_c - P_h^{Dc}(A)h^{Dc} - P^{X\widetilde{c}}(A)h^{X\widetilde{c}}] \tag{10}$$

[10] We suppress the state arguments of the pricing functions in (11) and elsewhere below to simplify the equations.

$$h^{Dc} = (\theta_{Hc})^{\rho} \left(\frac{P_h^{Dc}}{P_c}\right)^{-\rho} H_c \text{ and } h^{X\tilde{c}} = (1-\theta_{Hc})^{\rho} \left(\frac{P_h^{X\tilde{c}}}{P_c}\right)^{-\rho} H_c \tag{11}$$

These conditional demands and technology together yield the aggregate price level.

$$P_c(A) = \left[(\theta_{Hc})^{\rho} \left(P_h^{Dc}(A)\right)^{1-\rho} + (1-\theta_{Hc})^{\rho} \left(P_h^{X\tilde{c}}(A)\right)^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{12}$$

trade part 1: Country 1's real imports of final goods inputs (hereafter, *h-goods*) are $\frac{P_h^{X2}}{P_1}\Psi_1 h^{X2}$, in units of country 1 final output. Its real exports of h-goods are $\frac{P_h^{X1}Q}{P_2}\Psi_2 h^{X1}$, or $\frac{P_h^{D1}}{P_1}\Psi_2 h^{X1}$ given the *law of one price* and the real exchange rate Q. Country 2's real h-good imports are $\frac{P_h^{X1}}{P_2}\Psi_2 h^{X1}$, in units of country 2 final output, and its real h-good exports are $\frac{P_h^{X2}}{P_1 Q}\Psi_1 h^{X2} = \frac{P_h^{D2}}{P_2}\Psi_1 h^{X2}$.

3.2.2 Material goods retailers

Production and trade in the material goods sector is as in the final goods sector, though we allow home bias parameters to differ across sectors. The material goods retailer in country c uses per-capita domestic wholesale goods, m^{Dc}, and exports from country $\tilde{c}$, $m^{X\tilde{c}}$, in the production function below to produce per-capita materials, M_c.

$$M_c = \left[\theta_{Mc}\left(m^{Dc}\right)^{\frac{\rho-1}{\rho}} + (1-\theta_{Mc})\left(m^{X\tilde{c}}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}, \tag{13}$$

It sells the materials it produces at price $P_c^M(A)$ to domestic manufacturing firms, and it pays $P_m^{Dc}(A)$ and $P_m^{X\tilde{c}}(A)$ for its domestic and foreign wholesale inputs (hereafter, *m-goods*). Given these nominal prices and the technology in (13), the country c materials retailer solves the problem in equation 14, with resulting per-capita conditional factor demands in (15).

$$\max_{m^{Dc}, m^{X\tilde{c}}} \Psi_c[P_c^M(A)M_c - P_m^{Dc}(A)m^{Dc} - P_m^{X\tilde{c}}(A)m^{X\tilde{c}}] \tag{14}$$

$$m^{Dc} = (\theta_{Mc})^{\rho}\left(\frac{P_m^{Dc}}{P_c^M}\right)^{-\rho} M_c \text{ and } m^{X\tilde{c}} = (1-\theta_{Mc})^{\rho}\left(\frac{P_m^{X\tilde{c}}}{P_c^M}\right)^{-\rho} M_c \tag{15}$$

Combining equations (13) and (15), we have the nominal price of materials.

$$P_c^M(A) = \left[(\theta_{Mc})^{\rho}\left(P_m^{Dc}(A)\right)^{1-\rho} + (1-\theta_{Mc})^{\rho}\left(P_m^{X\tilde{c}}(A)\right)^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{16}$$

trade part 2: Real imports and exports of m-goods in country 1 are, respectively, $\frac{P_m^{X2}}{P_1}\Psi_1 m^{X2}$ and $\frac{P_m^{X1}Q}{P_2}\Psi_2 m^{X1} = \frac{P_m^{D1}}{P_1}\Psi_2 m^{X1}$. Country 2's real imports and exports of m-goods are $\frac{P_m^{X1}}{P_2}\Psi_2 m^{X1}$ and $\frac{P_m^{X2}}{P_1 Q}\Psi_1 m^{X2} = \frac{P_m^{D2}}{P_2}\Psi_1 m^{X2}$. Combining trade for use in this sector with that for use in the final goods sector above, we have per-capita net exports for each country's real GDP accounts.

$$NX_c = \frac{\Psi_{\tilde{c}}}{\Psi_c}\left(\frac{P_h^{Dc}}{P_c}h^{Xc} + \frac{P_m^{Dc}}{P_c}m^{Xc}\right) - \left(\frac{P_h^{X\tilde{c}}}{P_c}h^{X\tilde{c}} + \frac{P_m^{X\tilde{c}}}{P_c}m^{X\tilde{c}}\right) \tag{17}$$

3.3 Wholesalers

The representative wholesaler in each country c combines manufactured goods, Y_c, and labor, N_c^w, to supply h-goods and m-goods to retailers at home and abroad. It is indifferent to where its goods are sold, as the law-of-one-price prevails; $\frac{P_i^{X1}Q}{P_2} = \frac{P_i^{D1}}{P_1}$ and $\frac{P_i^{X2}}{P_1 Q} = \frac{P_i^{D2}}{P_2}$ for $i = h, m$. We furthermore assume the same Leontief technology for producing h-goods and m-goods,

$$W_c = \min\left\{Y_c, \frac{N_c^w}{a_N}\right\}, \text{ where } a_N > 0, \tag{18}$$

implying the same marginal cost of production and hence common real price for h- and m- goods.

Denoting the real price of country c domestic manufactured goods by $\omega_{yc}(A)$ and its per-capita supply of h- and m- goods by W_{hc} and W_{mc} respectively, the wholesaler's problem is:

$$\max_{W_{hc}, W_{mc}} \Psi_c\Big[\frac{P_h^{Dc}(A)}{P_c(A)}W_{hc} + \frac{P_m^{Dc}(A)}{P_c(A)}W_{mc} - \omega_{yc}(A)[W_{hc} + W_{mc}] - a_N w_c(A)[W_{hc} + W_{mc}]\Big]. \tag{19}$$

Its first order conditions imply these restrictions on the prices of wholesale and manufactured goods.

$$P_m^{Dc} = P_h^{Dc} \equiv P^{Dc} \tag{20}$$

$$\omega_{yc} = P^{Dc}/P_c - a_N w_c \tag{21}$$

3.4 Manufacturing firms

Manufacturing sectors are also symmetric across countries, so we describe them at once in this section using country identifier c. Reviewing notation from above, country c firms receive real price $\omega_{yc}(A)$ for their output, and they pay real wage $w_c(A)$ for labor and real price $\frac{P_c^M(A)}{P_c(A)}$ for materials. Their capital investments are in units of country c final goods.

Each firm enters a period identified by $s \equiv (k, b, \varepsilon)$, where k and b are the capital and debt levels it selected at the end of last period, and ε is its current persistent idiosyncratic productivity. Positive values of b represent debt; negative values are financial savings. The firm repays its debt (or recoups its savings) and produces using capital, labor and materials in a decreasing returns to scale Cobb-Douglas production function:

$$y = z_c \varepsilon k^\alpha n^\nu m^\gamma, \tag{22}$$

where z_c is the aggregate productivity shock in its country, $\alpha \in (0,1)$, $\nu \in (0,1)$, $\gamma \in (0,1)$, and $\alpha + \nu + \gamma < 1$. Like aggregate productivity, firm-specific productivity ε follows a Markov chain; it has N_ε realizations and evolves according to transition probabilities $\varphi_{ij}^\varepsilon = pr(\varepsilon' = \varepsilon_j \mid \varepsilon = \varepsilon_i)$.

After production, each firm realizes the outcome of a state-invariant, exogenous exit shock. At that point, fraction $\chi \in (0,1)$ of firms exit the economy with $k' = b' = 0$. Each exiting firm undertakes a negative investment of size $(1-\delta)k$ and returns its cash as dividends to domestic households as it departs. Continuing incumbent firms choose their future capital stocks and debt ($b' > 0$) or financial savings ($b' \leq 0$) subject to a collateral constraint and non-negative dividends. Exiting firms are replaced at the start of the next period by an equal number of new firms that each begin with capital stock k_{0c}, debt b_{0c} and a productivity drawn from the ergodic distribution of ε; thus the total investment in new firms each period is χk_{0c}.

3.4.1 static choices, profits and cash

Given its capital and productivity, the domestic real wage, real materials cost and relative price of its output, each firm chooses its labor and materials inputs to solve the following static problem, subject to the production technology in (22).

$$\max_{n,m} \omega_{yc}(A) y(n,m;k,\varepsilon;A) - w_c(A)n - \frac{P_c^M(A)}{P_c(A)} m \tag{23}$$

The firm's labor and materials decision rules, and thus its output supply, follow immediately. That in turn recovers its static profits, $\pi_c(k,\varepsilon;A) = (1-\nu-\gamma)\omega_{yc} y_c(k,\varepsilon;A)$, the real value of sales less flow input costs. Note that each of these is independent of the firm's financial position.

$$n_c(k,\varepsilon;A) = \left[\gamma z_c \varepsilon \omega_{yc} \left(\frac{\nu}{\gamma w_c} \right)^{1-\gamma} \left(\frac{P_c^M}{P_c} \right)^{-\gamma} k^\alpha \right]^{\frac{1}{1-\nu-\gamma}} \tag{24}$$

$$m_c(k,\varepsilon;A) = \gamma w_c \left(\nu \frac{P_c^M}{P_c} \right)^{-1} n_c(k,\varepsilon;A) \tag{25}$$

Consolidating terms and defining a summary variable, $\Omega_c(A)$, we can write profits as a multiplicative function of aggregate versus individual states in (26), which will be useful for determining the capital decision rules below.

$$\pi_c(k,\varepsilon;A) = (1-\nu-\gamma)\Omega_c(A) \left[\varepsilon k^\alpha \right]^{\frac{1}{1-\nu-\gamma}} \tag{26}$$

$$\text{where } \Omega_c(A) \equiv \left[z_c \omega_{yc} \gamma^\gamma \left(\frac{P_c^M}{P_c} \right)^{-\gamma} \left(\frac{\nu}{w_c} \right)^\nu \right]^{\frac{1}{1-\nu-\gamma}} \tag{27}$$

Finally, let x represent the (k,b,ε) firm's real cash-on-hand, the sum of its static profit and non-depreciated capital net of outstanding debt:

$$x_c(k,b,\varepsilon;A) \equiv \pi_c(k,\varepsilon;A) + (1-\delta)k - b. \tag{28}$$

Exiting firms return their x to households as they leave the economy; continuing firms use it to finance capital investment and/or dividends, solving the intertemporal problem below.

3.4.2 intertemporal choices

A continuing firm receives $q_c(A)$ units of domestic final goods in the current period for each unit of debt it incurs; thus, a debt with face value b' delivers a real loan of size $q_c(A)b'$. Capital accumulation is one period time-to-build; $k' = (1-\delta)k + i$, where i is investment. The firm's choices of k', b' and current dividends, D, are thus restricted by the budget constraint:

$$x_c(k, b, \varepsilon; A) + q_c(A)b' \geq D + k'. \tag{29}$$

We assume the firm cannot issue new equity to finance its investment, $D \geq 0$, and the debt it takes on is limited in proportion to its existing capital by the collateral constraint: $b' \leq \zeta_c(A)k$, where $\zeta_c \geq 0$ is the exogenous state variable reflecting the current availability of credit in country c.

Continuing firms make decisions taking as given the equilibrium evolution of the endogenous aggregate state, $S' = \Gamma(S, Z)$, and the transition probabilities π^Z_{lm} governing the evolution of the exogenous state Z. There are no real frictions impeding capital adjustments, so we can use the profit function (26) to explicitly solve for *target capitals* $k^*_c(\varepsilon, A)$ selected in absence of financial constraints. These ε-specific targets will facilitate our solution for firms' capital decisions.

We impose state-contingent discounting consistent with equilibrium in the market for shares (section 3.1) in stating firms' intertemporal problem. Let $\Lambda_c(A)$ be the value a country c firm ascribes to dividends, where $A = (S, Z)$, and assume firms discount their future value by the household discount factor β. In equilibrium, $\Lambda_c(A)$ is the domestic household marginal utility of consumption, so we are simply expressing the firm value function in units of marginal utility.

Let v^e_c represent the value of a country c firm just prior to the realization of the exit shock:

$$v^e_c(k, b, \varepsilon; A) = \chi \Lambda_c(A) x_c(k, b, \varepsilon; A) + (1-\chi) v_c(k, b, \varepsilon; A), \tag{30}$$

where v_c is the expected discounted value conditional on continuing to the next period. Because the dividends of a continuing firm are immediate as a function of its k', b' choice from the binding budget constraint (29), we can write the problem of a continuing firm of type (k, b, ε_i) as follows.

$$\begin{aligned} v_c(k, b, \varepsilon_i; S, Z_l) \;=\; & \max_{k', b'} \Big[\Lambda_c(S, Z_l)[x_c(k, b, \varepsilon; S, Z_l) + q_c(S, Z_l)b' - k'] \\ & + \beta \sum_{m=1}^{N_Z} \sum_{j=1}^{N_\varepsilon} \pi^Z_{lm} \varphi^\varepsilon_{ij} v^e_c(k', b', \varepsilon_j; S', Z_m) \Big] \end{aligned} \tag{31}$$

$$\text{subject to} \quad : \quad b' \leq \zeta_c(A)k \;\; \text{and} \;\; x_c(k, b, \varepsilon; A) + q_c(A)b' - k' \geq 0 \tag{32}$$

The second constraint in (32) prevents the firm from using negative dividends to evade the first.

The problem above is simplified further by these observations. In equilibrium, no continuing firm can strictly raise its value by paying positive dividends, since it borrows and lends at the same price its owners face, and $\Lambda_c(A) = \lambda_c(A)$. If a firm has amassed sufficient cash to prevent its investment ever again being hindered by inadequate funds, its shadow value of retained earnings matches the household valuation of dividends. Any such *impervious* firm invests efficiently, and it is indifferent about paying dividends. On the other hand, for a firm with insufficient cash to preclude the possibility that the collateral constraint (32) may bind in some future state, the per-unit valuation of retained earnings exceeds the domestic household's dividend valuation. Any such financially *exposed* firm must set $D = 0$ to maximize its value, and its investment may be affected by its cash. With this classification, it is straightforward to recover the firm decision rules below.

Impervious firm decision rules: As there are no *real* frictions impeding capital adjustment, an impervious firm maximizes its value by adopting the efficient (target) capital $k_c^*(\varepsilon_i, \cdot)$ satisfying:

$$\Lambda_c(S, Z_l) = \beta \sum_{m=1}^{N_Z} \pi_{lm}^Z \Lambda_c(S', Z_m) \left[1 + \delta + \sum_{j=1}^{N_\varepsilon} \varphi_{ij}^\varepsilon \partial \pi_c(k', \varepsilon_j; S', Z_m)/\partial k' \right].$$

Thus, given $\Omega_c(A)$ in (27) and $S' = \Gamma(A)$, the $i = 1, \ldots, N_\varepsilon$ impervious firm capital choices follow.

$$k_c^*(\varepsilon_i, S, Z_l) = \left[\frac{\alpha}{1 - (1-\delta) q_c(S, Z_l)} \sum_{m=1}^{N_Z} \pi_{lm}^Z \frac{\beta \Lambda_c(S', Z_m)}{\Lambda_c(S, Z_l)} \Omega_c(S', Z_m) \sum_{j=1}^{N_\varepsilon} \varphi_{ij}^\varepsilon \varepsilon_j^{\frac{1}{1-\nu-\gamma}} \right]^{\frac{1-\nu-\gamma}{1-\alpha-\nu-\gamma}} \quad (33)$$

By definition, an impervious firm is indifferent about dividends. So long as we ensure it remains so, we can choose how the indeterminacy is resolved. One natural approach is to assume any such firm pays zero dividends, $D_c^Z(k, b, \varepsilon, A) = 0$, and retrieve its corresponding debt rule from the budget constraint: $B_c^Z(k, b, \varepsilon, A) = [k_c^*(\varepsilon_i, A) - x_c(k, b, \varepsilon; A)]/q_c(A)$. Another natural approach is to follow Khan and Thomas (2013) in assigning any such firm a *minimum savings* debt policy ensuring that it will have sufficient funds to implement its optimal investment plan without borrowing more than is permitted by (32) under all possible futures, and then retrieve its dividends from the budget constraint. We adopt a version of the latter approach here to avoid an ever-extending left tail in the debt distribution, though our results change in no substantive way under the former approach.[11]

[11] Given our model's large aggregate state vector and number of relative prices, we study impulse responses here rather than the full stochastic solution. Thus, the information set for our minimum savings policy differs from that in Khan and Thomas (2013). Our solution for the steady-state policy presumes ignorance of aggregate shocks; whereas, our solution in each date of the impulse responses presumes knowledge of the next date's Z. As a result, not all firms identified as impervious in our steady-state remain so throughout our impulse responses; we confirm this alters no real results by verifying that all such firms nonetheless implement efficient investment in every date of the impulse.

The minimum savings policy $B_c(\varepsilon_i, S, Z_l)$ solves (34) - (35); $\widetilde{B}_c\Big(k_c^*(\varepsilon_i, S, Z_l), \varepsilon_j; S', Z_m\Big)$ is the largest debt a firm can hold, alongside capital $k_c^*(\varepsilon_i, S, Z_l)$, and still be impervious entering next period if the exogenous state is (ε_j, Z_m). The minimum $\widetilde{B}_c(\cdot)$ over all possible (ε_j, Z_m) is the largest debt with which the firm can exit *this* period and know it will be impervious next period, $B_c(\varepsilon_i; A)$.

$$B_c(\varepsilon_i, S, Z_l) = \min_{\{\varepsilon_j | \varphi_{ij}^{\varepsilon} > 0 \text{ and } Z_m | \pi_{lm}^{Z} > 0\}} \widetilde{B}_c\Big(k_c^*(\varepsilon_i, S, Z_l), \varepsilon_j; \Gamma(S, Z_l), Z_m\Big) \tag{34}$$

$$\widetilde{B}_c(k, \varepsilon; A) = \pi_c\,(k, \varepsilon; A) + (1-\delta)k - k_c^*(\varepsilon; A) + q_c(A)\min\{B_c(\varepsilon; A), \zeta_c(A)k\} \tag{35}$$

Equation 35 is the start of period maximum debt under which a firm can adopt its target capital and a debt not exceeding that identified by the minimum savings policy. This is the b at which adopting impervious firm decision rules implies $D = 0$. The minimum operator imposes the collateral constraint while still identifying a firm borrowing only $\zeta_c(A)k < B_c^I(\varepsilon; A)$ as impervious if it has sufficient cash to finance its investment. Finally, the impervious firm capital and debt policies, $g_c^I(k, b, \varepsilon; A) = k_c^*(\varepsilon; A)$ and $f_c^I(k, b, \varepsilon; A) = \min\{B_c^I(\varepsilon; A), \zeta_c(A)k\}$, together imply dividends:

$$D_c^I(k, b, \varepsilon; A) = \pi_c(k, \varepsilon; A) + (1-\delta)k - b - k_c^*(\varepsilon; A) + q_c(A)\min\{B_c^I(\varepsilon; A), \zeta_c(A)k\}. \tag{36}$$

Categorizing firms: We can easily tell if any (k, b, ε) firm is impervious from the policies above. Suppose the firm adopts $k' = g_c^I(k, b, \varepsilon; A)$ and $b' = f_c^I(k, b, \varepsilon; A)$. If this implies $D_c^I(k, b, \varepsilon; A) \geq 0$, it is impervious and adopts the posited policies. Otherwise, it is still exposed to financial market frictions and so adopts the policies we describe below.

Exposed firm decision rules: As an *exposed* firm always sets $D = 0$, its capital choice implies its debt through the budget constraint (29): $f_c^E(k, b, \varepsilon; A) = [g_c^E(k, b, \varepsilon; A) - x_c(k, b, \varepsilon; A)]/q_c(A)$. Since b' is bounded by the collateral constraint, this in turn bounds its feasible capital choice: $k' \leq x_c(k, b, \varepsilon; A) + q_c(A)\zeta_c(A)k$. If the firm can reach the efficient capital $k_c^*(\varepsilon; A)$ without violating this bound, it adopts it. Otherwise, given the strictly concave profit function, it chooses the nearest k' it can afford. Thus, the exposed firm capital rule is simply:

$$g_c^E(k, b, \varepsilon; A) = \min\{k_c^*(\varepsilon; A), x_c(k, b, \varepsilon; A) + q_c(A)\zeta_c(A)k\}. \tag{37}$$

3.5 Recursive equilibrium

Let $\widetilde{c}$ represent country c's trade partner; $\widetilde{c} \neq c$. A *recursive competitive equilibrium* is a set of functions: $\varrho, Q, \{w_c, q_c, \rho_c, \widetilde{\rho}_c, P_c\}_{c=1,2}$, $\{P_h^{Dc}, P_h^{X\widetilde{c}}, P_m^{Dc}, P_m^{X\widetilde{c}}, \omega_{yc}\}_{c=1,2}$, $\{V_c^h, C_c, N_c, \xi_c', \phi_c', B_c'\}_{c=1,2}$, $\{H_c, h^{Dc}, h^{X\widetilde{c}}, M_c, m^{Dc}, m^{X\widetilde{c}}\}_{c=1,2}$, $\{W_{hc}, W_{mc}, Y_c, N_c^w\}_{c=1,2}$, and $\{n_c, m_c, y_c, g_c, f_c\}_{c=1,2}$ that solve household and firm problems and clear the markets for assets, final goods, labor, materials, and wholesale and manufactured goods in each country, as outlined by the following conditions.

(i) V_1^h solves (1) and V_2^h solves (6), with associated policy functions $(C_c, N_c, \xi_c', \phi_c', B_c^{'})$ for $c = 1, 2$

(ii) final goods retailers solve (10), with policy functions $(H_c, h^{Dc}, h^{X\widetilde{c}})$ for $c = 1, 2$

(ii) material goods retailers solve (14), with policy functions $(M_c, m^{Dc}, m^{X\widetilde{c}})$ for $c = 1, 2$

(iii) wholesalers solve (19), with policy functions $(W_{hc}, W_{mc}, Y_c, N_c^w)$ for $c = 1, 2$

(iv) manufacturing firms (k, b, ε) solve (23) with policy functions (n_c, m_c, y_c) for $c = 1, 2$

(v) v_c solves (30) - (31) subject to (32), with associated policy functions (g_c, f_c), for $c = 1, 2$

(vi) $\xi_c'(k', b', \varepsilon_j, \xi_c, \phi_c, B_c; A) = \mu_c'(k', b', \varepsilon_j; A)$, for each $(k', b', \varepsilon_j) \in \mathcal{S}$ in country $c = 1, 2$

(vii) $\phi_c'(\xi_c, \phi_c, B_c; A) = \int f_c\,(k, b, \varepsilon; A)\,\mu_c(d\,[k \times b \times \varepsilon])$, for $c = 1, 2$

(viii) $\Psi_1 B_1'(A', \xi_1, \phi_1, B_1; A) + \Psi_2 B_2'(A', \xi_2, \phi_2, B_2; A) = 0$ for all $(A'; A)$

(ix) $\Psi_c C_c(\xi_c, \phi_c, B_c; A) = \Psi_c H_c(A) - \Psi_c I_c(A)$, where:

$$\Psi_c I_c(A) \equiv \int [(1-\chi) g_c(k, b, \varepsilon; A) + \chi k_{0c} - (1-\delta)k] \mu_c(d\,[k \times b \times \varepsilon]), \text{ for } c = 1, 2$$

(x) $\Psi_c N_c(\xi_c, \phi_c, B_c; A) = \Psi_c N_c^w(A) + \int n_c(k, \varepsilon; A) \mu_c(d\,[k \times b \times \varepsilon])$, for $c = 1, 2$

(xi) $\Psi_c M_c(A) = \int m_c(k, \varepsilon; A) \mu_c(d\,[k \times b \times \varepsilon])$, for $c = 1, 2$

(xii) $\Psi_c W_{hc}(A) = \Psi_c h^{Dc}(A) + \Psi_{\widetilde{c}} h^{X\widetilde{c}}(A)$, for $c = 1, 2$

(xiii) $\Psi_c W_{mc}(A) = \Psi_c m^{Dc}(A) + \Psi_{\widetilde{c}} m^{X\widetilde{c}}(A)$, for $c = 1, 2$

(xiv) $\Psi_c[W_{hc}(A) + W_{mc}(A)] = \Psi_c N_c^w(A)/a_N = \Psi_c Y_c(A)$, where:

$$\Psi_c Y_c(A) = \int y_c(k, b, \varepsilon; A) \mu_c(d\,[k \times b \times \varepsilon]), \text{ for } c = 1, 2$$

(xv) $\mu_c'(\mp, \varepsilon_j) = (1-\chi) \int_{\{(k,b,\varepsilon_i)\ |\ (g_c(k,b,\varepsilon_i;A), b_c'(k,b,\varepsilon_i;A)) \in \mp\}} \varphi_{ij}^{\varepsilon} \mu_c(d\,[k \times b \times \varepsilon_i]) + \chi \mathcal{J}(k_0) M(\varepsilon_j)$,

$\forall (\mp, \varepsilon_j) \in \mathcal{S}$, defines Γ, where $\mathcal{J}(k_0) = \{1$ if $(k_0, 0) \in \mp$ and 0 otherwise$\}$, for $c = 1, 2$

GDP: We end this section with each country's real per-capita GDP in units of its own final goods, starting from the expenditure side: $GDP_c = C_c + I_c + NX_c$. Using items (ix) and (xiv) above with the NX_c equation (17) from section 3.2, price restrictions from section 3.3 and conditional factor demands from section 3.2, we can write this as a relative price weighted function of total production less material inputs in the manufacturing sector: $GDP_c = [\omega_{yc} + a_N w_c]Y_c - \frac{P_c^M}{P_c} M_c$. Alternatively, $GDP_c = [(1-\gamma)\omega_{yc} + a_N w_c]Y_c$, after aggregating firms' static first order conditions in section (3.4).

4 Calibration

The length of a period in our model is one quarter. We normalize country 1's population to 1 and calibrate its parameters using annualized moments drawn from postwar U.S. data. The only parameters differing in country 2 are those associated with its size and trade shares. We select these to reflect Canada's relative size and trading shares with the United States, as described below.

We assume households have period utility: $u(C, L) = \frac{1}{1-\phi}\left(\left[C - \frac{\kappa}{\eta}(1-L)^{\eta}\right]^{1-\phi} - 1\right)$, adopting the preferences of Greenwood, Hercowitz and Huffman (1988). This specification is widely used in international business cycle models because it removes consumption from the intra-temporal marginal rate of substitution, thereby eliminating wealth effects on labor supply.[12] We set the household discount factor β to obtain a long-run annual real interest rate of 4 percent consistent with measurement in Gomme, Ravikumar and Rupert (2011). Next, we fix $\eta = 1.25$ to imply a labor supply elasticity of 4, and set the coefficient of relative risk aversion at $\phi = 2$ as in Backus, Kehoe and Kydland (1994), Kehoe and Perri (2002), Alessandria and Choi (2007) and elsewhere.

The capital depreciation rate δ is set to yield an average annual aggregate investment-to-capital ratio at roughly 7 percent, consistent with that for the private capital stock between 1954 and 2002 in the U.S. Fixed Asset Tables, controlling for growth. We choose χ so that 9 percent of firms exit each year, guided by the 0.087 average among firms in the Bureau of Labor Statistics' Business Dynamics Statistics database (BDS) over 1979 - 2007, and we fix the materials share parameter γ at 0.43 to match the average materials share of manufacturing in the U.S. input-output tables from the World Input-Output Database over 2000-2014.[13] The remaining model parameters are jointly calibrated, though our discussion below will link parameters with the targets they most influence.

Capital's share in manufacturing, α, is chosen to reproduce the 2.34 average annual private

[12] See, for example, Devereux, Gregory and Smith (1992), Schmitt-Grohé and Uribe (2003), Raffo (2008), and Alessandria, Kaboski and Midrigan (2013). Raffo (2008) shows its use in a standard two-country business cycle model delivers the observed countercyclical net flow of goods across countries.

[13] Our average reflects all sectors other than aggriculture and commodities, and is similar to that for Canada, 0.48.

capital-to-GDP ratio in the U.S. over 1954-2017. Labor share parameters in the manufacturing and wholesale sectors, ν and a_N, are taken to imply labor's share of GDP is 0.6 and wholesale trade's value added share of GDP matching the U.S. average over 1997-2019, at 5.97 percent. Next, we set the labor disutility parameter κ and country 1's steady state productivity z_1^* so that its steady state GDP is 1, with households each working one-third of available time.

TABLE 1. Parameter values

(a) Population and preferences, retail production, mean aggregate tfp and credit states

Ψ_2	β	ϕ	η	κ	ρ	θ_{h1}	θ_{m1}	θ_{h2}	θ_{m2}	z_1^*	z_2^*	ζ^*
0.064	0.990	2	1.25	2.4	0.9	0.994	0.989	0.898	0.896	2.113	2.875	0.508

(b) Wholesale and mfg production, death and birth, depreciation and idiosyncratic tfp

a_N	α	ν	γ	χ	k_{01}	b_{01}	k_{02}	b_{02}	δ	ρ_ε	σ_e
0.018	0.178	0.237	0.430	0.023	1.494	0.598	1.467	0.587	0.018	0.901	0.03

NOTE.– Quarterly frequency; all parameter values rounded to nearest 0.001.

Country 2's relative population size Ψ_2 and steady state productivity z_2^* are taken to ensure its GDP averages 8.8 percent that in country 1, the average Canada-to-U.S. ratio over 1994-2007, while per-capita labor hours match those in country 1. Turning to trade-related parameters, we set the elasticity of substitution between domestic and imported goods ρ at 0.9, as estimated by Heathcote and Perri (2002).[14] Next, we choose the four parameters reflecting home bias in each country's materials and final goods sectors, $\{\theta_{mc}, \theta_{hc}\}_{c=1,2}$, so that our model's steady state reproduces average U.S. real imports from Canada relative to GDP (0.021) and Canadian real imports from the U.S. relative to GDP (0.207) over 1994-2007 from the International Monetary Fund's Direction of Trade Statistics, alongside average U.S. materials inputs imported from Canada relative to total U.S. materials inputs use (0.017) and Canadian materials inputs imported from the U.S. relative to total Canadian materials inputs use (0.127) over 2000-2007.[15]

We turn now to idiosyncratic productivity, ordinary credit terms, and the initial conditions of new manufacturing firms. Productivities are drawn from a log-normal distribution with persistence ρ_ε and standard deviation σ_ε, discretized using $N_\varepsilon = 7$ values. We set ρ_ε to imply a 0.659 annual

[14] Corsetti et al. (2008) estimate the elasticity of substitution between home and foreign tradeables at 0.85 through the lens of a two-country model with tradeable and non-tradeable goods; there, the U.S. represents the home country and the trade-weighted aggregate of Canada, Japan and EU-15 is the foreign country.

[15] The North American Free Trade Agreement took effect in 1994; hence our choice in computing average import shares. Our source for materials input shares, The World Input-Output Tables database, is available from 2000.

persistence as in Khan and Thomas (2013). Next, our σ_ε ensures that a comparable sample of firms in our model's steady state reproduces the 0.337 average cross-sectional standard deviation of annual investment rates documented by Cooper and Haltiwanger (2006) using panel data from the Longitudinal Research Database. Finally, we calibrate ζ^* and $\{k_{0c}, b_{0c}\}_{c=1,2}$ so the debt-to-asset ratio of new firms in each country is 40 percent (Kauffman Firm Survey) and targeting the BDS average employment sizes of new and 1-year-old firms relative to incumbent firms over 1990-2006, 27 and 36 percent, respectively. While not expressly targeted in the calibration, our model's average employment share of young firms (aged 0 - 5 years) is 19 percent, versus 17 percent in the BDS.

5 Results

Our purpose in this study is to analyze how a financial recession in a large, developed country like the United States affects its trading partners purely through trade channels. Given its proximity to the U.S. in both geographic and development terms, the fact that it is a leading trading partner for the U.S. and the fact that the U.S. is by far its lead trading partner, we have calibrated our model's second country to Canada despite its small size. We begin this section by considering the overall outcomes in country 1 (U.S.) and country 2 (Canada) when they experience a credit shock simultaneously, and demonstrating the underlying mechanics in either country's response to its domestic shock. Next, we study the trade-based transmission of the U.S. shock to Canada through two counterfactual exercises. The first compares country 2's recession in the calibrated global crisis to that absent any shock in country 1; the second considers the responses in country 2 absent any shock in country 2. Finally, we examine international transmission of productivity shocks through a similar set of exercises to understand what aspects of credit shock transmission are unique.

5.1 Financial shocks

We begin our analysis of financial shocks by exploring a simultaneous tightening of collateral constraints in country 1 and country 2, lowering ζ_1 and ζ_2 relative to their steady state values. We choose the size and persistence of the two shocks so that the dynamic paths of debt in the two countries are consistent with those in the United States (country 1) and Canada (country 2) during the global financial crisis of 2007-09. Specifically, we lower ζ_1 by 31.48 percent and ζ_2 by 14.26 percent, and we hold them fixed at these levels for 8 quarters. Thereafter, in date 9, ζ_1 and ζ_2 begin reverting to their normal values according to an AR(1) process with persistence 0.939. Given this path of shocks, we see a peak-to-trough decline in country 1 debt of 25.9 percent over

9 quarters, and the series is still 20.9 percent below its starting point a year later (in date 13), as happened in the United States. Meantime, country 2 debt falls by 11.4 percent over the first 9 quarters, and is still 9.33 percent below steady state one year later, as observed in Canada.[16]

FIGURE 1. Responses to simultaneous credit shocks in countries 1 and 2

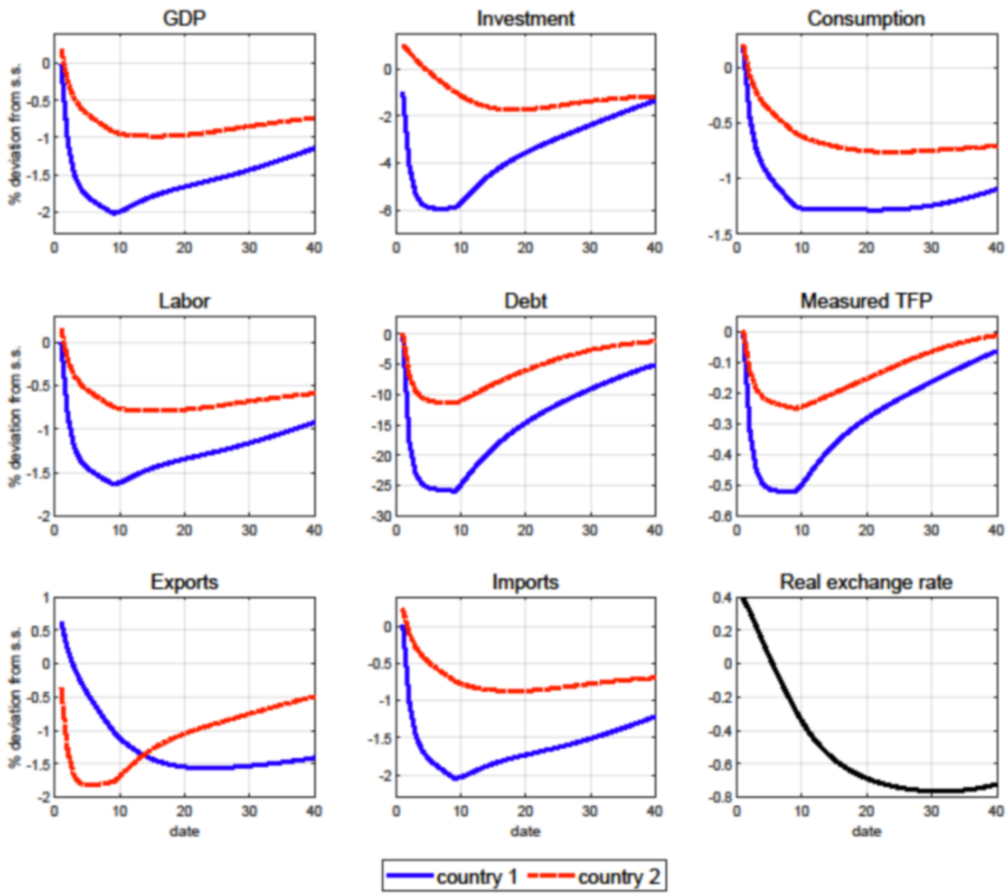


Figure 1 presents aggregate responses in country 1 and country 2 to the simultaneous credit shocks described above. Recall that GDP in each country is a price-weighted function of total production among manufacturing firms, $GDP_c = [(1-\gamma)\omega_{yc} + a_N w_c]Y_c$, and it is these firms that are subject to collateral constraints. As collateral constraints tighten in date 1, restricting firms' ability to take on debt to finance their investments, we see GDP, investment and hours worked (labor) declining in country 1, pulling consumption down with a slight lag. These three series reach their trough after 8 quarters, in date 9, when credit conditions begin to improve.[17]

[16] Our debt series from the U.S. is commercial and industrial loans, deflated by GDP deflator; our Canadian series is loans to non-financial corporations and unincorporated businesses, deflated by GDP deflator.

[17] Our peak-to-trough declines in country 1 GDP and investment are smaller than in Khan and Thomas' (2013) closed economy study for three main reasons. First, country 1 new firms have capital stocks roughly 16 percent of the

Country 2's responses are similar to those in country 1. It also experiences a persistent recession, though a milder one given its smaller credit shock. During the initial periods of contraction, both countries cut their imports from abroad, reacting to contractions in domestic demand. Country 1's imports fall equally for final goods and material goods inputs (see appendix figure A1), whereas country 2's import decline involves a greater reduction in material goods inputs. Another notable distinction in the country 2 recession is the fact that its GDP and hours worked series rise in the first quarter of the shock, and its investment is above average for a full year. We trace the reasons for these differences to international trade linkages below in section 5.2, but first focus here on the common aspects driven by each country's domestic credit shock.

An endogenous decline in productivity (figure 1, row 2) drives the persistent contraction in each country. We use figures 2 and 3 to discuss the rising misallocation underlying this decline, here focusing our analysis on country 1.[18] As with other heterogeneous producer studies featuring financial shocks, aggregate productivity falls over an episode of tight credit in our model because the allocation of production moves further from the efficient one implied by firms' productivities. This is in turn driven by increased distortions in the allocation of capital across firms, with disruptions to the firm life cycle featuring heavily in our setting as in Khan and Thomas (2013).

When an aggregate shock tightens collateral constraints, some firms that otherwise would not face binding borrowing limits impeding their investments in physical capital now do, while those that would have faced such limits now find their activities more constrained than otherwise. These problems do not affect firms evenly or orthogonally, of course; those hardest hit are cash-poor firms most reliant on external funds for their investments, whereas larger firms capable of self-financing efficient investment experience no ill effects. Because young firms tend to be cash-poor and rely on external funds to help them grow to their mature sizes, their investments are disproportionately hindered by a credit shock. Young firms draw from the same productivity distribution as do other firms, so this disparate influence on their capital choices necessarily increases overall misallocation, lowering aggregate productivity. Moreover, looking within any cohort among firms with binding collateral constraints, those with relatively high productivity tend to be more adversely affected.

Figure 2 shows how the world credit crisis affects the allocation of production in country 1, though these disaggregated pictures would be the same without the country 2 shock. Panel (a)

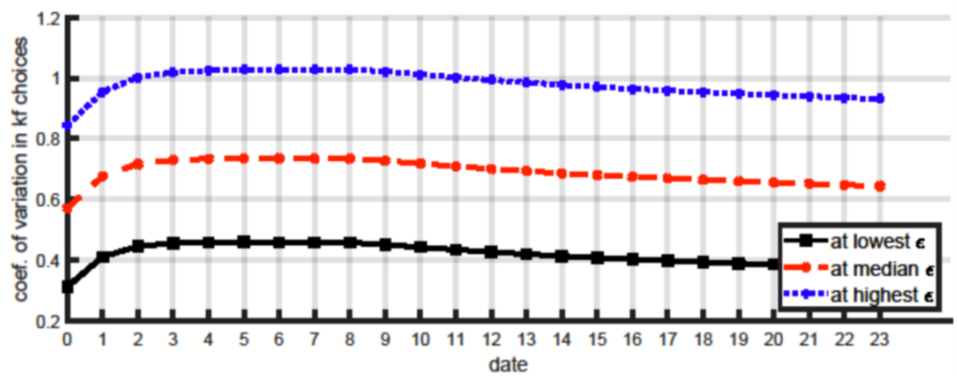
average stock in our calibration, whereas Khan and Thomas' entrants had 10 percent relative capital size and so were more reliant on debt and for longer. Second, our exit rate is 1 percent lower, implying fewer firms in the maturing phase most hurt by financial tightening. Third, our country 1 credit shock is smaller and shorter: a 16 (versus 88) percentage point reduction in ζ_1 held in place for 2 (versus 4) years before mean reversion begins.

[18] The same forces play out in country 2, as may be seen from figure A3 in the appendix.

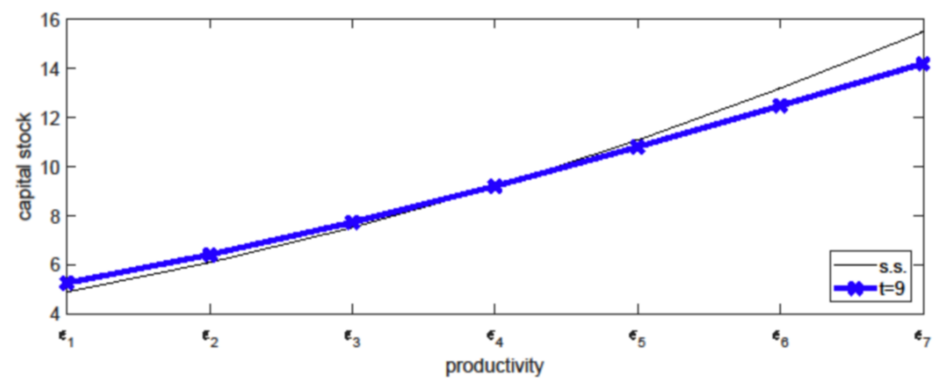
presents the date-by-date coefficient of variation in capital choices among firms drawing the lowest, median and highest idiosyncratic productivity levels starting from the shock's impact in date 1, with the steady state counterparts plotted at date 0. Absent financial frictions, each firm's capital choice would depend only on its current productivity and aggregate prices; in other words, efficient investment dictates that all firms with a common current productivity level adopt the same capital for the next period, implying zero coefficients of variation conditional on productivity. In our model economy, however, binding collateral constraints prevent some firms from reaching their efficient capitals, moving these coefficients away from zero. Notice that, in steady state (date 0) and at each date of the impulse, this measure of investment inefficiency rises with idiosyncratic productivity, as the gap between affordable and efficient capital widens for cash-poor firms.

FIGURE 2. Inefficiency measures in country 1

(a) Coefficient of variation in capital choices



(b) Average capital stocks in steady state versus recession trough date



NOTE.– Panel (a) plots variation in capital choices conditional on productivity; date 0: steady state; date 9: GDP trough date. Panel (b) plots average capital at each productivity level in steady state and at the GDP trough date.

The dispersion in firms' capital choices rises at all productivity levels with the tightening of credit in quarter 1. These inefficiency measures rise further for a few periods as the number of young firms with their life cycles disrupted grows with the birth of new cohorts, and they remain elevated until quarter 9 while collateral values are held low. Thereafter, as credit conditions begin improving, the coefficients of variation slowly revert toward their ordinary values. This persistently raised inefficiency in capital choices reduces aggregate productivity and gradualizes its recovery, which in turn leads to protracted contractions in investment, employment and GDP.

While we saw in panel (a) that steady state inefficiency rises in idiosyncratic productivity, panel (b) further suggests that the *rise* in capital misallocation brought on by aggregate credit shocks is greater among firms with higher idiosyncratic productivity levels. Comparing the average capital stock among firms with each idiosyncratic productivity at the GDP trough date versus in steady state, note that the average capital among the highest productivity firms falls further below normal than happens among firms with productivities near the median. Meanwhile, the average stocks among firms with the lowest productivities actually rise slightly. This seemingly peculiar outcome is brought on by the falling real interest rate that naturally accompanies a misallocation-induced decline in aggregate productivity. Given a lowered real cost of investment and no change in exogenous aggregate productivity, efficient capital choices rise for firms at all idiosyncratic productivity levels.[19] Thus, those with adequate cash to reach their target stocks invest more than they would in ordinary times. Given the mean reverting productivity process, the proportion of such firms is greater at lower productivity levels where efficient capital stocks are smaller, so we see a slight rise in the average stock for these groups. On balance, the credit shock tilts the distribution of capital, and so production, toward less productive firms, contributing to lowered aggregate productivity.

Figure 3 demonstrates that the misallocation induced by credit shocks particularly affects young firms. Because they are more reliant on debt than other firms, credit shocks protract the maturing phase over which young firms gradually build up debt and capital to reach sizes consistent with their productivities. Panel (a) shows the paths of debt (in blue) and capital (in black) for the mean firm in a cohort born in steady state (dotted lines) versus in a cohort born when the credit shocks hit. Following each of these cohorts over their first 19 quarters, we see that the typical firm born in the credit shock recession accumulates debt and hence capital more gradually than does the typical firm born in steady state. In its 10th quarter of life, the average firm born at the date of the shock has only 53 percent the debt it would have were it born in steady-state.

Panel (a) establishes that young firms born during a credit crisis are persistently smaller than they otherwise would be. However, as the overall economy shrinks in response to credit shocks, it does not prove that these firms are more affected than others. Panel (b) more directly considers

[19] Appendix figure A2 plots each country's real interest rate and target capital associated with median productivity.

the extent of misallocation over the early ages of our two young cohorts, plotting the average excess return to capital (the gap between the expected discounted marginal value of adopted capital versus its unit purchase price) at each age. Comparing the cohort born at the date of the shocks (solid blue) to the one born in steady state (dotted blue), note that firms born in the credit crisis have larger credit-induced wedges between their optimal capitals and the capitals they adopt than they would in ordinary times. This elevated inefficiency is most pronounced in their earliest ages when they have little capital to borrow against and collateral requirements are unusually high. However, it persists well beyond quarter 8, when credit conditions start improving. In fact, this cohort does not invest as efficiently as it would in steady state until it is about 9 years old.

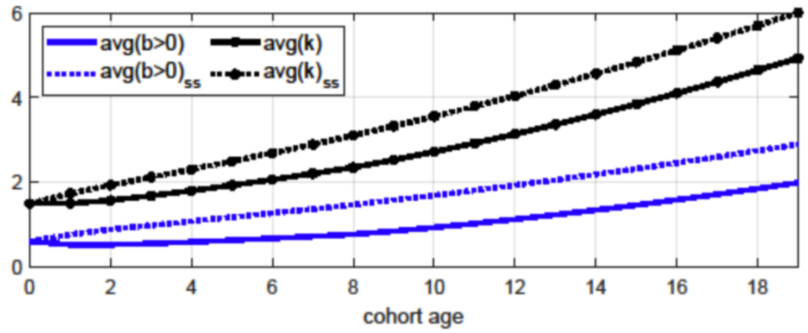
Panel (c) shows how the disparate rise in inefficiency noted above affects the relative sizes of young firms, reinforcing the observation that increased capital misallocation is more pronounced among young firms during credit shocks. There, we plot per-member average capital relative to the economywide average stock in our two young cohorts over their first 19 quarters. Consistent with the discussion of panel (b), we see that the shock-born cohort (in solid black) has persistently lower relative capital, and hence a persistently lower productive share, than it would experience over its maturing phase in ordinary times (dotted black). For example, 18 quarters after birth, the typical firm in the shock-born cohort has a capital stock roughly 50.4 percent of the economywide average; had it been born in steady state, its relative capital size at the same age would have been 61 percent.[20] In sum, although credit shocks reduce investment and production overall, we see that young firms make up a disproportionate share of these declines.

While figure 3 illustrates persistently elevated investment inefficiency only for the cohort born at the impact date of the credit recession, the same problems arise for every young cohort so long as credit is tight. As we alluded to in explaining panel (a) of figure 2, each cohort born during the credit crisis experiences a similarly slowed growth phase. Because firms born in dates just prior to the shock also take longer to reach maturity than they would otherwise, this for a time increases the number of cohorts making their way toward maturity, and thus the overall fraction of firms investing inefficiently, compounding the problem of increased inefficiency for each such firm. Even after credit conditions fully recover, it takes considerable time for the distribution of capital to recover its normal shape after such persistent disruptions to the growth paths of cumulated cohorts. This slows the recovery in aggregate productivity in figure 1, and thus the real interest rate and domestic savings, thereby protracting the recoveries in GDP, employment and consumption.

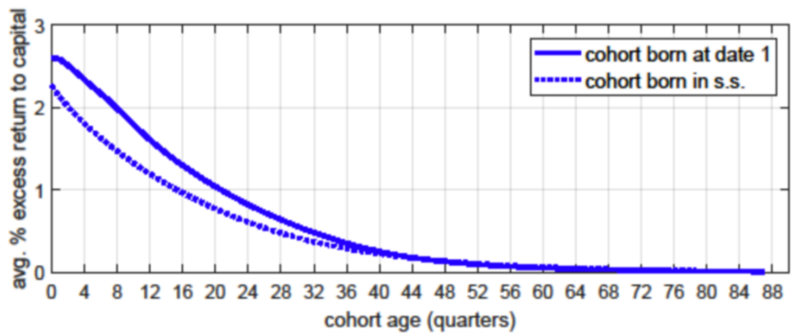
[20] Appendix figure A3 shows similar delays in the growth of young firms in country 2, for the same reasons.

FIGURE 3: Country-1 cohort born at shock impact versus steady state

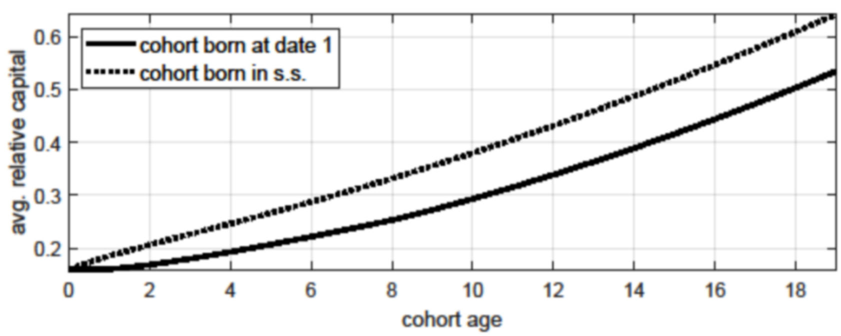
(a) Average debt and average capital



(b) Investment inefficiency



(c) Average relative capital

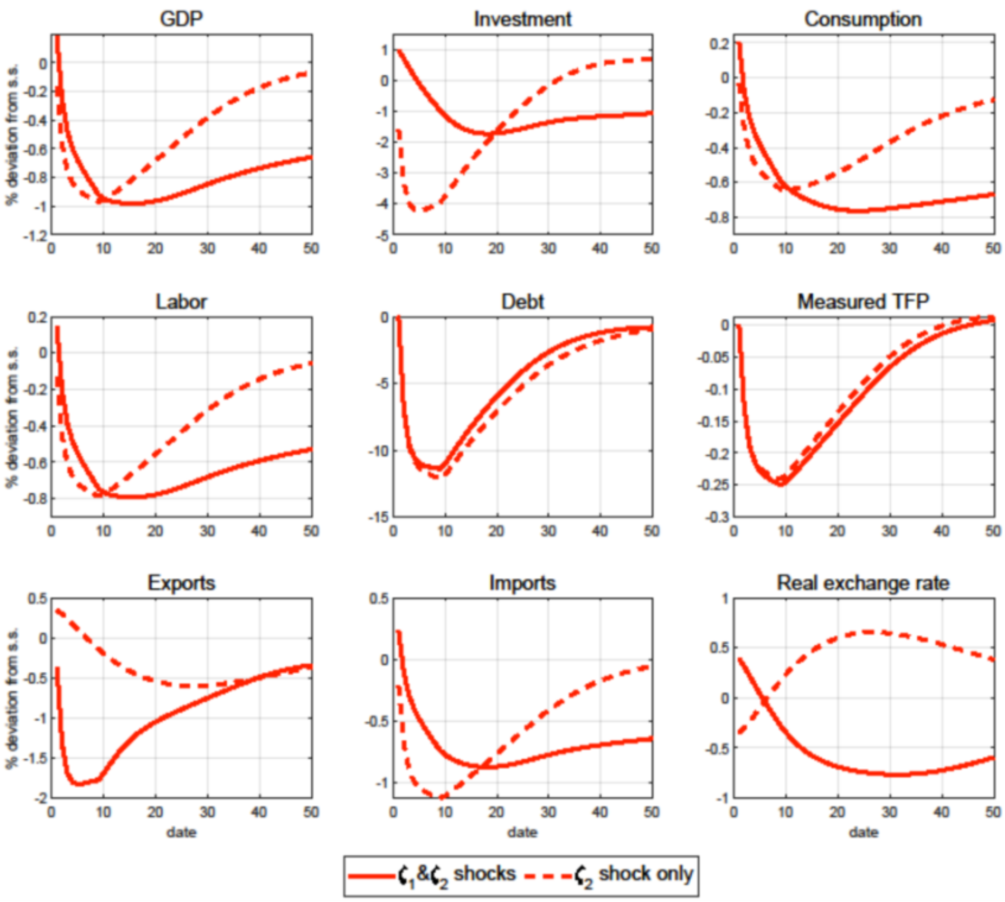


5.2 International transmission of credit shocks

While section 5.1 emphasized some common mechanics underlying the two countries' initially similar responses to the simultaneous credit shocks in figure 1, we noted that the country 2 downturn is distinguished from that in country 1 by raised employment and GDP in the first quarter of the shock and by a few quarters of above average investment. Other, more striking differences appear later in the figure 1 responses, however, where we see the recession trough in country 2 is delayed and its subsequent recovery considerably more gradual relative to experiences in country 1.

This section traces the persistence of country 2's contraction to the recessionary effects of the credit shock in its large trade partner propagating across borders through international trade. As we discussed in section 1, the global economy experienced a dramatic synchronization in business cycles during and following the 2007 U.S. financial crisis. Taking as given that financial contagion was instrumental in driving the business cycle synchronization over this period, our results suggest that international goods trade also may have played an important role in generating persistent stagnation in small open economies and delaying global recovery following the crisis.

FIGURE 4. International transmission of a country 1 credit shock



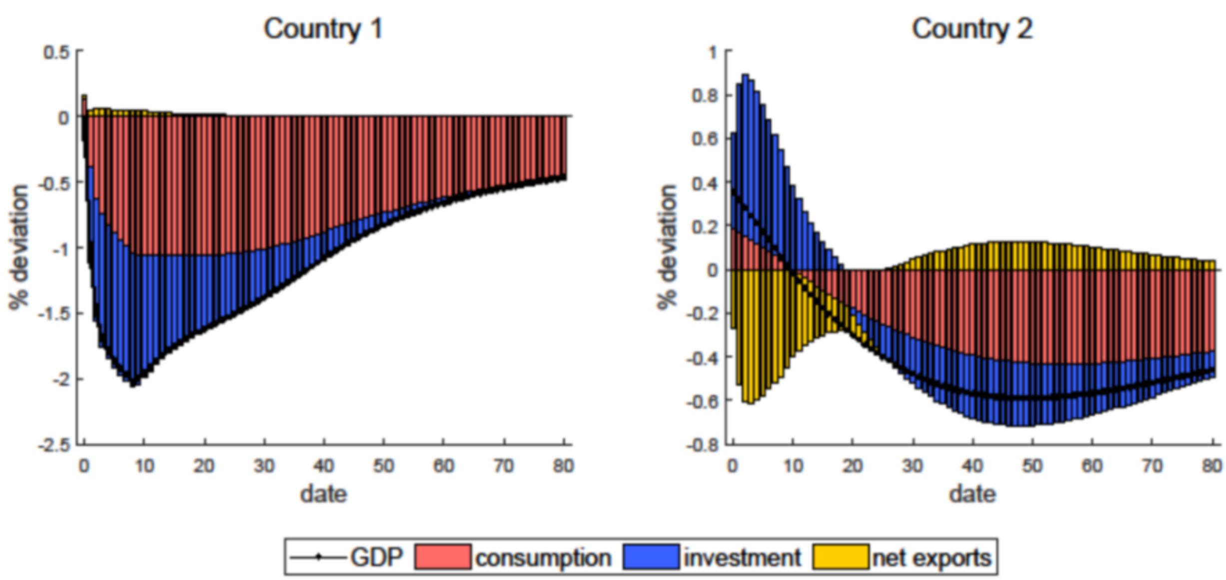
NOTE.– Country 2 responses to credit shocks in both countries (solid lines) versus only in country 2 (dashed lines).

Figure 4 isolates which aspects of our small open country's financial recession and recovery stem from its international trade with a much larger country also experiencing a financial recession. Here, we compare country 2's responses to a domestic credit shock (dashed lines) with its responses when its U.S. trading partner also suffers a credit shock (solid lines), maintaining the same shock sizes and persistence as in section 5.1. We interpret the gaps between solid and dashed lines in each panel as reflecting the transmitted effects of country 1's credit crisis to country 2 through trade.[21]

When country 2 alone is hit by a credit shock, its responses resemble those of country 1 from figure 1. There, we see large declines in investment and debt from the onset of the shock, and GDP steadily declines until quarter 9 when the credit shock begins to dissipate. Thereafter, as overall borrowing recovers, measured productivity and so other aggregate variables gradually recover. One notable difference in comparison with figure 1 is a considerably milder response in country 2 exports. Given the country's relative size, its credit shock has little effect on the country 1 economy, so movements in country 2 exports simply reflect the changes in the real exchange rate.

When both countries have credit shocks, the transmission of country 1's shock alters the country 2 recession in three clear ways. First, the downturn is milder over its first two years, particularly in the earliest few quarters; for example, GDP is down roughly half as much in date 2 as happens without a country 1 shock. Second, it lasts longer, with the GDP trough date occurring in quarter 15 rather than 9. Third, and most strikingly, the recovery phase is far more gradual. Measuring from their respective trough dates, the half-life of the GDP response is extended by a decade, from 5.5 to 15.5 years. Focusing on the most directly affected series, investment is 1.3 percent below normal 32 quarters out, where it would have recovered fully absent the country 1 credit shock.

FIGURE 5. Decomposition of GDP response to a country 1 credit shock



[21] Appendix figure A4 presents country 1 responses to simultaneous credit shocks versus a domestic shock only, revealing negligible transmission from country 2 to country 1. Recall that country 2 GDP is only 8.8 percent that in country 1 and country 1's imports from country 2 are equivalent to only 2.2 percent of its GDP in our calibration.

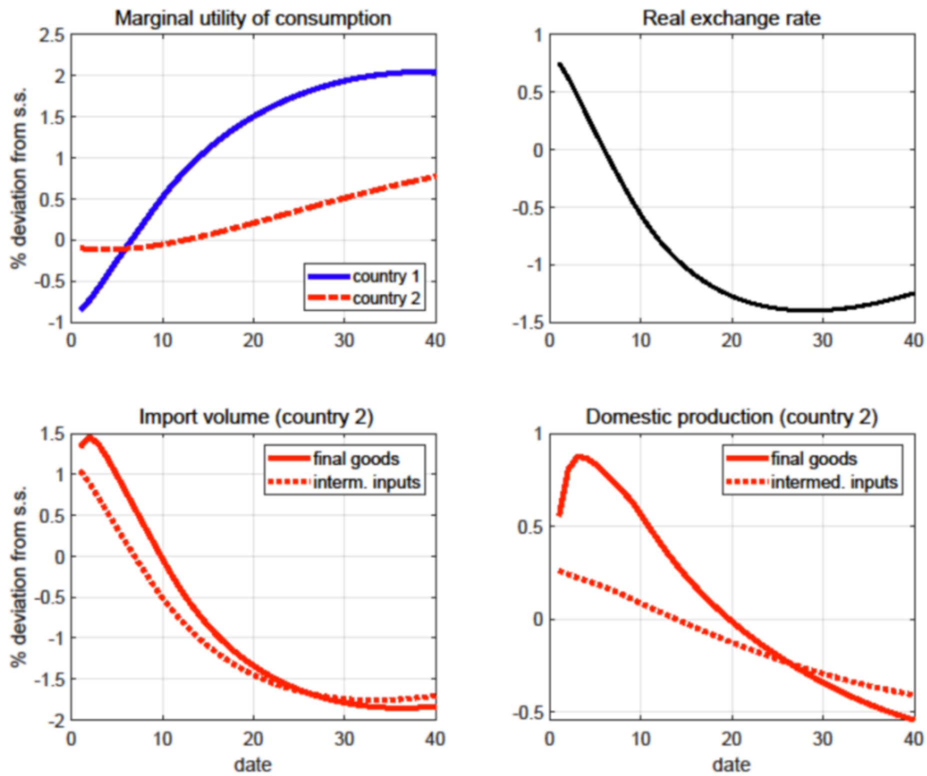
We disentangle the dynamics of credit shock transmission by focusing now on the effects in each country arising from a country 1 credit shock in isolation. Figure 5 displays the GDP responses to the country 1 credit shock, decomposing it into the contributions of its three components: consumption, investment and net exports. The domestic credit shock in country 1 (at left) induces declines in consumption and investment over the first 8 quarters of tight credit and then slow recovery over many subsequent periods, just as we saw in figure 1. Declines in consumption and investment account for roughly equal shares of the total fall in GDP over the downturn, while consumption contributes an increasingly large share in the recovery phase. Net exports for country 1 play only a marginal role here, given the small country 1 trade share of GDP in our calibration.

The same country 1 credit shock induces an immediate, temporary rise in country 2's GDP (at right), absent any direct shock affecting country 2. The initial GDP rise in country 2 is partly accounted for by a brief rise in consumption, but mostly stems from a large rise in investment. Together, these initial rises more than offset the fall in net exports accompanying reduced demand for its exports in country 1. Soon thereafter, though, country 2 consumption and investment begin long declines into levels below steady state. Country 2 GDP finally reaches its trough in quarter 50, and the subsequent recoveries in consumption and investment are strikingly gradual.

Figure 6 explores the mechanics of country 2's investment response to the country 1 credit shock. Households in country 1 temporarily raise their consumption in response to the domestic credit shock, anticipating lowered returns to saving driven by falling aggregate productivity. This reduces the marginal utility of consumption in country 1 relative to that in country 2 (top left), causing a real appreciation of the country 2 currency (top right). For country 2, the resulting fall in the real cost of goods produced in country 1 drives raised imports of final goods and materials inputs (bottom left). Recalling that goods from the two countries are complementary inputs in our calibration, that in turn raises demand for domestically produced final goods and intermediate inputs, stimulating investment in physical capital required for their production.

We saw in figure 5 that country 2's initial rises in investment and consumption are temporary. The turnaround in consumption coincides with that of the real exchange rate in figure 6. Country 1's currency begins appreciating relative to steady state after date 6, as the economic downturn deepens and consumption there enters a fairly steep decline. At that point, the weakened value of the country 2 currency begins reducing demand for imports from country 1, reversing the input channel discussed just above and compounding the effects of low demand for country 2 exports in country 1. There follows a long contraction in county 2, with investment leading the decline as production adjusts to an extended episode of lowered demand, domestic and abroad.

FIGURE 6. Responses to a credit shock directly affecting only country 1



5.3 International transmission of productivity shocks

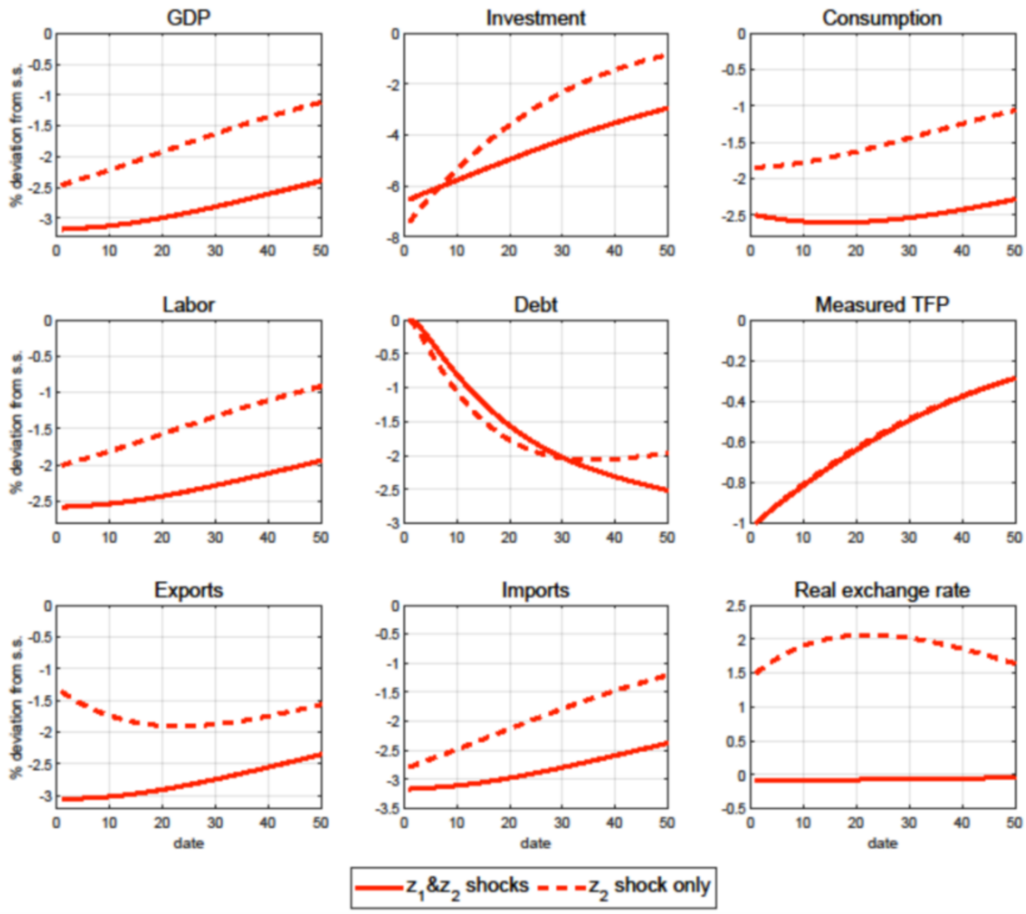
Having studied the domestic effects and international transmission of credit shocks, we close with a look at productivity shock transmission, considering how its channels relate to those analyzed above. Figure 7 presents country 2's responses to 1 percent negative productivity shocks followed by AR(1) recovery with persistence 0.977. Analogous to our approach in section 5.2, we compare how country 2 responds to a domestic TFP shock only (dashed lines) versus simultaneous shocks in both countries (solid lines), and we interpret the differences between series in each panel as reflecting how the productivity shock in country 1 affects country 2 through international trade.

Unlike the responses to credit shocks in figures 1 and 4, there are no gaps between the paths of measured TFP and the productivity shocks themselves in figure 7. This mirrors the finding in Khan and Thomas (2013), and arises for the same reason here as there. Whereas figures 2 and 3 showed that credit shocks disproportionately depress investments of young, growing firms and those with high relative productivity, thereby amplifying misallocation, aggregate productivity shocks have no such disparate effects. Production and target capital stocks scale down evenly across firms in proportion to their usual productive shares, implying no change in the distribution of production.[22]

[22] Supporting figures are available upon request.

Our second, more novel observation from figure 7 is that the distinctions in how credit versus productivity shocks operate within a country extend to how they affect its trading partners. Unlike the delayed transmission of country 1's credit shock in figure 4, here we see its TFP shock lower GDP, employment and consumption in country 2 immediately. Moreover, whereas the effects of the country 1 credit shock for investment in country 2 dwarfed direct responses to its own shock, driving an initial rise in the series, delaying its decline and reducing it by more than half, the positive effects of the country 1 TFP shock for country 2's investment are modest and short-lived. The initial fall in the series is dampened by only about 1 percentage point for a few periods; thereafter, the effects of country 1's shock drag down country 2 investment throughout its recovery.

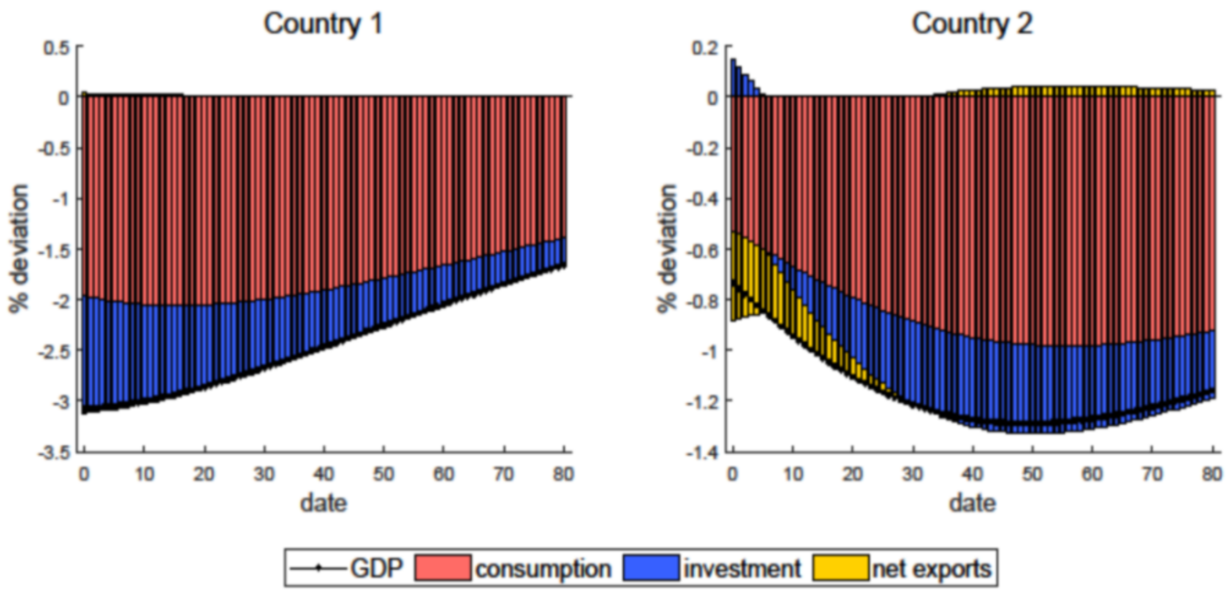
FIGURE 7. International transmission of a country 1 productivity shock



Given country 1's relative size and usual trade shares, its aggregate demand changes little in response to a TFP shock only in country 2. In that case, the fall in country 2 exports largely reflects movement in the real exchange rate driven by its recession, which raises the value of its currency and thus the real cost of its exports. When country 1 also has a productivity shock, however, the contraction in its large trading partner's aggregate demand depresses country 2 exports, while the exchange rate moves little given similar per-capita consumption declines in the two countries.

Figure 8 strips away the country 2 shock to directly examine how a productivity shock in country 1 affects domestic GDP (at left) and the GDP of country 2 (at right). In each panel, we decompose the response into its three expenditure-side components, as we did in figure 5 to study the transmission of a credit shock. Whereas investment had a central role in that exploration, the bulk of both GDP responses here are driven by consumption losses. In country 2, investment increases slightly during the initial few periods in response to the immediate rise in its relative productivity. Nonetheless, investment's share of GDP does not move far from its usual 16.7 percent, and the immediate contraction in consumption is a dominant factor driving the fall in GDP.

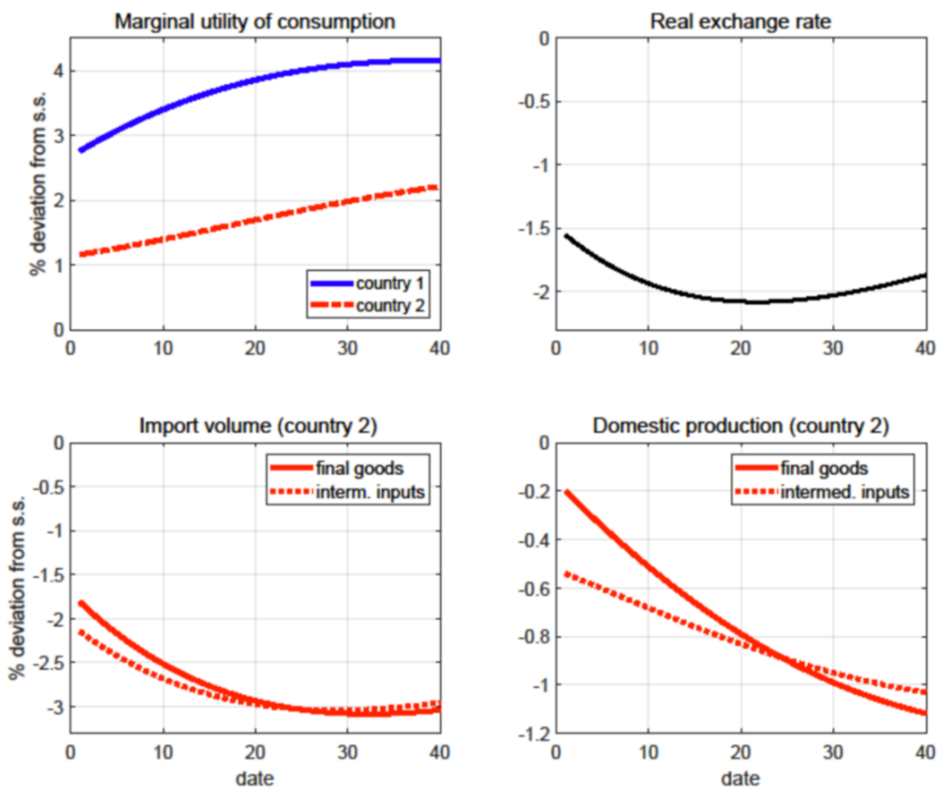
FIGURE 8. Decomposition of GDP response to a country 1 TFP shock



We saw in figure 5 that a credit shock in country 1 affects the domestic economy mainly through investment. Here, consumption losses make up roughly two-thirds of the initial domestic GDP decline in response to a country 1 productivity shock, with that share growing over time. Given the fall in its relative productivity at the onset of the shock, goods produced in country 1 grow more expensive, and we see the real exchange rate appreciate for country 1 in figure 9.

The consumption decline in country 1 following a TFP shock is considerably larger than in response to a financial shock, so consumption in country 2 can fall further without reversing the change in the exchange rate. The real depreciation of country 2's currency dampens its demand for imports from country 1, correspondingly lowering its demand for its own final goods and material inputs. On balance, because real versus financial shocks in country 1 affect the domestic economy through two different channels, necessary adjustments in the real exchange rate consistent with its relative productivity must also reflect those differing channels in the economy of its trading partner. Thus, the international transmission of foreign credit shocks occurs mainly through investment, whereas consumption has a more prominent role in the transmission of productivity shocks.

FIGURE 9. Responses to a TFP shock directly affecting only country 1



6 Concluding remarks

In this paper, we have developed a two-country equilibrium business cycle model to investigate the propagation of financial shocks and their transmission across countries through international goods trade. Our model features persistent heterogeneity in firm-level capital, productivity and collateralized debt, alongside input-output linkages within and across countries accommodating a sizeable role for trade in intermediate inputs. Its calibration to the U.S. and Canada is intended to reflect an epicenter of the global financial crisis interacting with one of its leading trade partners, but at the same time reflects the interactions of a large economy relatively impervious to trade with a smaller economy highly reliant on its external sector.

When confronted with simultaneous shocks to collateral constraints yielding paths of aggregate debt resembling those of overall business lending in the U.S. and Canada during the financial crisis, our model predicts large and persistent real economic downturns in the two economies, as increased misallocation of capital across firms generates endogenous declines in domestic aggregate productivity. Absent any other disruption, the resulting 2.02 and 0.98 percent peak-to-trough GDP declines account for roughly 40 and 20 percent of the observed declines in the U.S. and Canada. We show the main force underlying this raised misallocation is the fact that financial shocks disproportionately affect young firms, especially those with comparatively high productivities. Such firms are more reliant on external finance for their growth, and so suffer the largest deviations between the capital stocks they adopt and the efficient stocks consistent with their productivities.

In the face of global financial shocks, our model predicts that the large economy's recession is virtually unaltered by the shock to its smaller trade partner. By contrast, international transmission of the shock originating in the large economy substantively affects the small economy, delaying its recession trough and greatly extending the subsequent recovery phase. We interpret these results as an indication that, beyond the effects of financial contagion, international trade had a non-negligible role in the synchronization of business cycles across countries during the financial crisis and the prolonged episode of slow recoveries thereafter. Our model also predicts that investment dynamics have a prominent role in the transmission of credit shocks, whereas the transmission of productivity shocks both within and across countries happens mainly through adjustments in consumption. We take from this that the trade channel through which financial or real shocks most affect GDP responses abroad is principally reflective of their domestic transmission channel.

The analysis here might be usefully extended to explore recent findings on the disaggregated sources of the international trade collapse coinciding with the global financial crisis. Levchenko, Lewis and Tesar (2010) and Eaton, Kortum, Neiman and Romalis (2016) argue that the collapse came in large part from declines in expenditures on durable goods that are highly trade intensive. In particular, Eaton, Kortum, Neiman and Romalis (2016) use a business cycle accounting decomposition to identify the sources of the trade dynamics during the crisis; their results attribute two-thirds of the decline in trade relative to GDP during that period to shocks affecting the efficiency of investment in durable manufactures. As investment dynamics play the central role in propagating financial shocks in our model economy, a carefully calibrated extension including durable and nondurable goods sectors might be used to account for the decline in trade arising from a large shift in investment spending, shedding further light on real-financial linkages through international trade. We leave this exploration for future work.

Appendix: Additional figures

Figure A1 presents responses of real imports for country 1 and country 2 to simultaneous credit shocks in country 1 and country 2.

FIGURE A1. Impulse responses of real imports

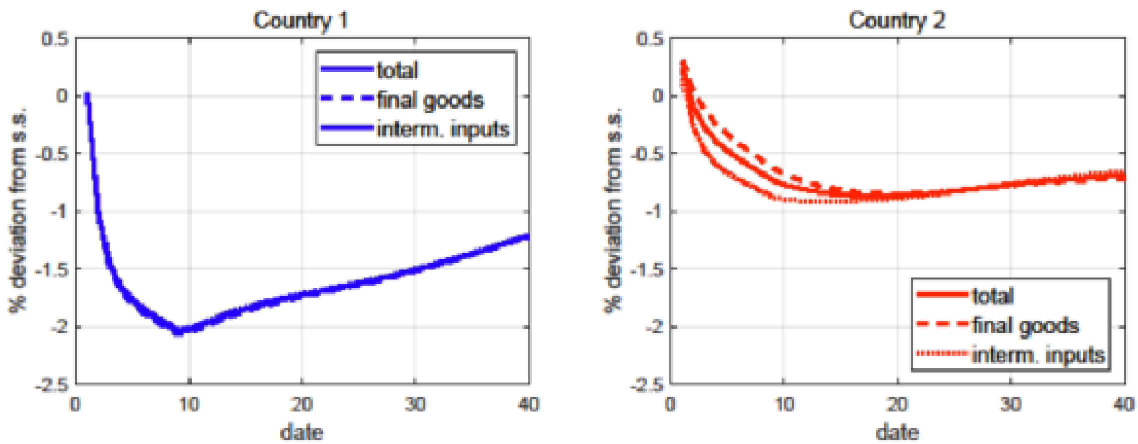


Figure A2 displays each country's corresponding changes in real interest rate, and in the efficient capital stock associated with median productivity.

FIGURE A2. Impulse responses

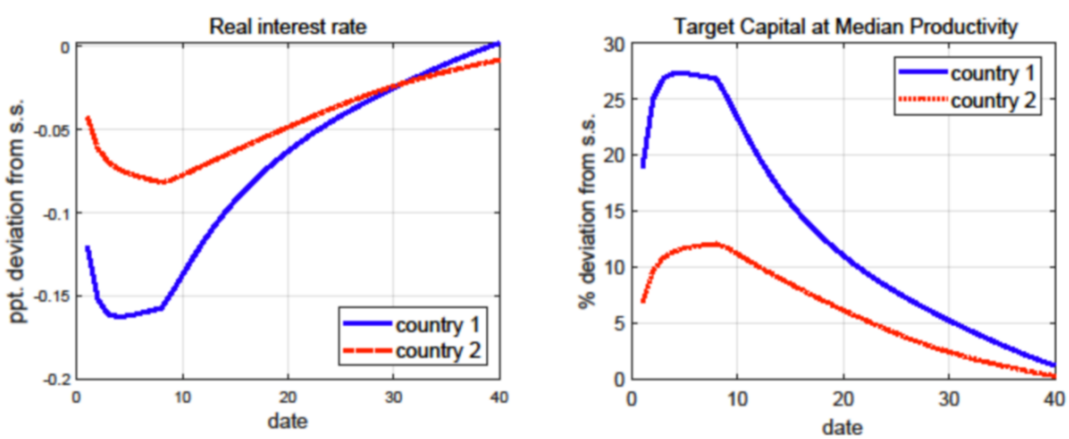
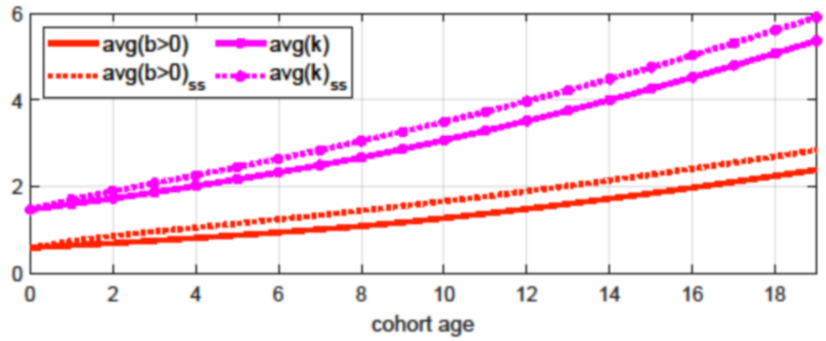


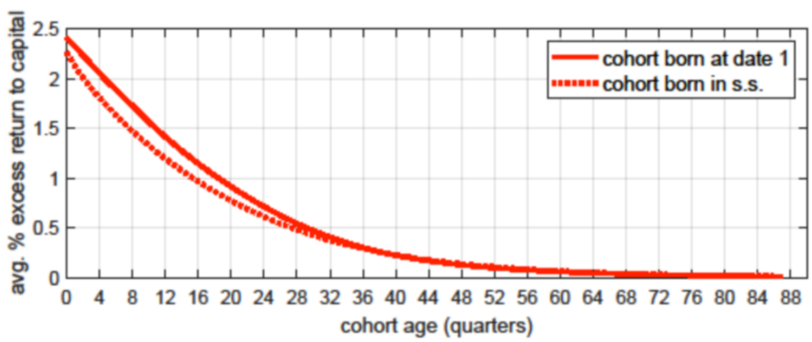
Figure A3 is the country 2 counterpart to figure 3 in the main text, comparing early quarters of life for a country 2 cohort born in steady state (dotted lines) to those for a cohort born at the date the credit shocks hit. Panel (a) shows the path of debt (in red) and capital (in pink) for the mean firm in the steady-state cohort and in the shock-date cohort. Panel (b) plots the average excess return to capital (the gap between capital's expected discounted marginal value and its unit purchase price) for the same two cohorts over many quarters, and panel (c) displays per-member average capital relative to the economywide average stock in their first 20 quarters.

FIGURE A3. Cohort born at shock impact versus steady state (country 2)

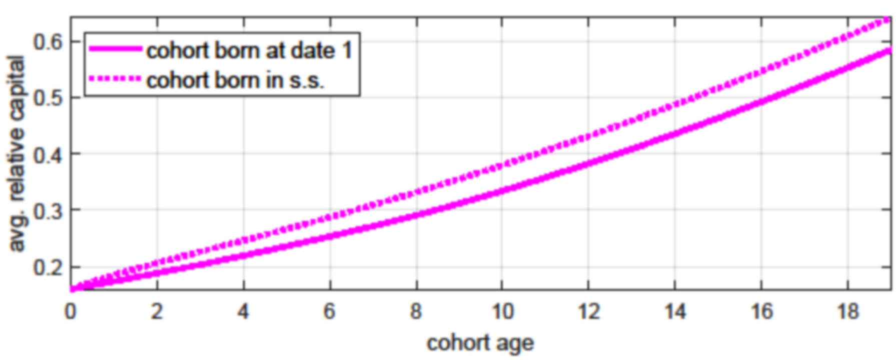
(a) Average debt and average capital



(b) Investment inefficiency

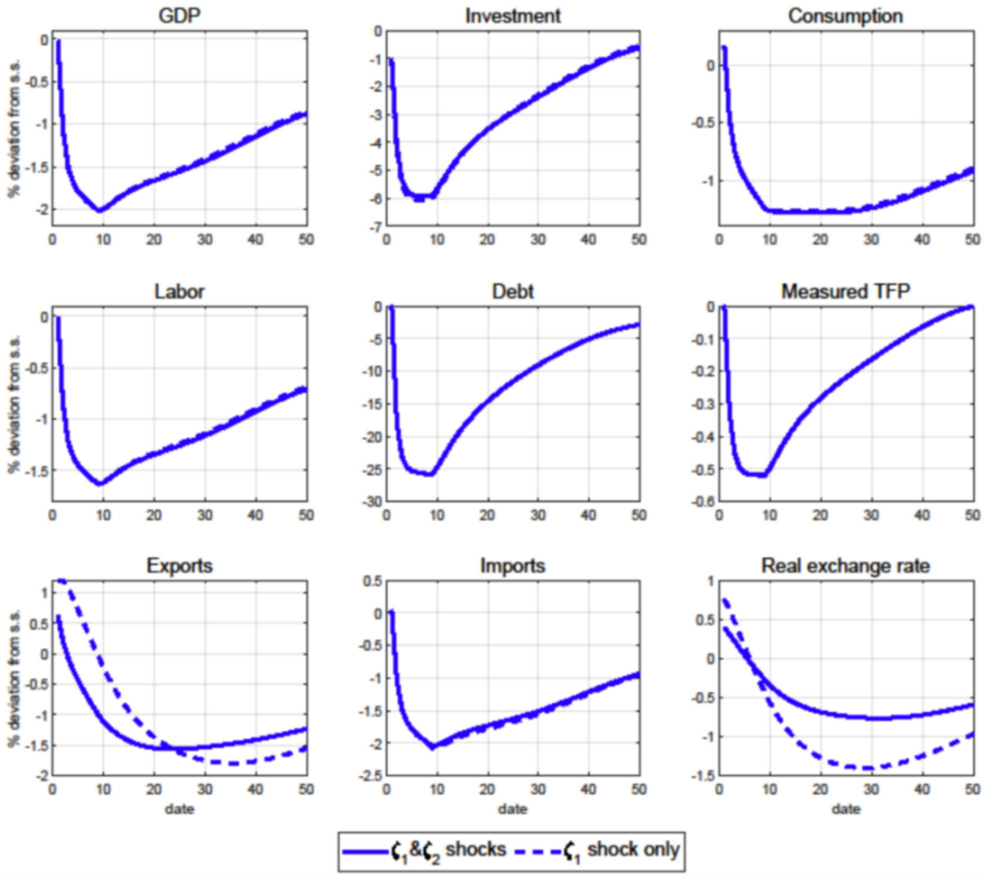


(c) Average relative capital



Finally, figure A4 compares the impulse responses in country 1 to simultaneous credit shocks in both country 1 and country 2 (blue solid lines), with those to a credit shocks in country 1 only (blue dashed lines). In each panel, the distance between the solid line and the dashed line represents the effects of a country 2 credit shock on country 1.

FIGURE A4. International transmission of a country 2 credit shock



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